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# Connection Establishment in LTE-A Networks: Justification of Poisson Process Modeling

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***Abstract*—The connection establishment in Long-Term Evolution Advanced (LTE-A) is often executed for distributed user equipment (UE) nodes with frequent small data sets for transmission to the central enhanced Node B. LTE-A connection establishment consists mainly of an access barring check (ABC) followed by preamble transmission (contention). Previous studies of connection establishment have often assumed Poisson characteristics (without verifying the Poisson assumption). In this paper, we introduce a simple equilibrium analysis framework for comprehensively evaluating the LTE-A connection establishment, including both access barring and preamble contention. We conduct a detailed analysis of the backlog arising from the uniform backoff over up to $T_o^{\max}$ slots by UE requests that failed the barring check or collided in the preamble contention. We verify that the process representing the numbers of backlogged UE requests rejoining the connection establishment tends to Poisson process characteristics for high barring probability and long maximum timeout $T_o^{\max}$. We present numerical comparisons of our equilibrium model with simulations for practical parameter settings. The comparisons illustrate the effects of the parameter settings on the convergence of the LTE-A connection establishment dynamics to Poisson characteristics for nonsynchronized and synchronized request arrivals.**

***Index Terms*—Access barring check (ABC), backlog model, Long-Term Evolution Advanced (LTE-A), Markov modulated poisson process, Poisson process, preamble contention, uniform backoff.**

## I. Introduction

THE Long-Term Evolution (LTE) and LTE-Advanced (LTE-A) standards developed by the Third-Generation Partnership Project (3GPP) are popular worldwide among cellular networks and hold promise to support a wide range of applications [1], [2]. For instance, LTE and LTE-A are considered viable for supporting emerging machine-to-machine (M2M) communications [3]–[5], which is also referred to as machine-type communications (MTC) [6], [7]. As an enabler for the futuristic Internet of Things (IoT), the performance and efficiency of MTC applications in LTE/LTE-A networks is an important research area.

MTC differs from conventional human-to-human communication in that MTC has small data sets (payloads) and high frequency of calls. Typically, an MTC client repeatedly accesses the server to send current status information or to query the server for status updates. In the context of LTE/LTE-A, the distributed UE nodes in a given cell gain access to the network through a central eNB. Due to relatively long idle times between successive UE transmissions, it is prudent that the UEs disconnect from the server and eNB until the next data set needs to be sent. This allows for significant statistical multiplexing, enabling support for a large number of UEs.

Since most UEs have to access the network for a very short duration, the success in gaining admission to the channel becomes a bottleneck. In the case of LTE/LTE-A, this bottleneck exists at accessing the eNB through the random access channel (RACH). The radio resource control (RRC) connection establishment (in brief "connection establishment") procedure in LTE/LTE-A RACH consists mainly of an access barring check (ABC), and the UEs clearing the ABC proceed to random preamble contention that is akin to single- or multiple-channel slotted Aloha random access (RA) [8]. In this paper, we comprehensively consider both the ABC and the preamble contention that together form the LTE/LTE-A connection establishment procedure.

Most prior analytical studies of LTE/LTE-A connection establishment assume Poisson characteristics for the random system quantities, such as the number of UE requests participating in the connection establishment procedure (ABC, followed by preamble contention for UEs clearing ABC) but do not validate the Poisson modeling. In contrast, in this paper, we closely examine the impact of the uniform backoff in LTE-A on the random system quantities. We show that the numbers of UE requests coming out of backoff and the numbers of UE requests participating in the connection establishment in the successive slots tend to Poisson processes for high barring probability and long maximum timeout $T_o^{\max}$.

In this paper, we make three main original contributions to the analysis of connection establishment in LTE/LTE-A networks.

1) We introduce a simple equilibrium analysis framework for evaluating the performance characteristics, such as success and drop probabilities of UE requests and throughput and delay, in RA systems operating with a

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single ($O = 1$) or multiple ($O > 1$) preambles. This analytical framework unifies the analysis of single-channel and multiple-channel multiple-access systems based on slotted Aloha principles.
2) We show that the uniform random backoff of barred or collided UE requests leads to Poisson characteristics of the numbers of UE requests in the connection establishment system. For a connection establishment system with high barring probability and long maximum timeout $T_o^{\max}$, we prove that the stochastic process representing the numbers of previously barred or collided requests (i.e., backlogged requests) that rejoin the connection establishment in successive slots tends to a Poisson process with a prescribed rate. Similarly, the numbers of UE requests that participate in the connection establishment procedures (ABC and, if not barred, preamble transmission) in successive slots approach a Poisson process.
3) Through numerical and simulation evaluations for a variety of request generation processes, including two-state Markov-modulated Poisson process arrivals, we demonstrate how ranges of parameter settings contribute to the simulated connection establishment dynamics approaching the analytical Poisson process model.

## II. Background and Related Work

The preamble contention in LTE-A is built on slotted Aloha principles but has a finite limit $W$ on the number of permitted transmission attempts. The dynamics of slotted Aloha with a limited number of transmission attempts, which are fundamentally different from the classical slotted Aloha [9], [10] with an unlimited number of transmission attempts, have been examined in [11]–[14]. It has been found that, up to a limit of eight transmission attempts, slotted Aloha systems have a single equilibrium operating point. Several studies have examined slotted-Aloha-based preamble contention in conjunction with mechanisms that resemble access barring. Seo and Leung [15] (and the closely related model by Yang *et al.* [16]) considered a persistence probability in their Markov-process-based analysis of backoff algorithms. However, in [15], barred UE requests retry the ABC in each subsequent slot. Sarker [17] considered a packet rejection probability, which is similar to ABC. However, the packet rejection in [17] is terminal, i.e., without retrial, whereas the LTE-A standard permits repetitive retrials of ABC.

A few studies have focused on access barring in LTE. For instance, cooperation among several cells to improve access barring performance has been examined in [18] and [19]. Within a given cell, a feedback control mechanism that selectively bars UE requests according to the congestion level has been proposed in [20], whereas load prediction to control barring has been examined in [21].

Some recent studies have specifically examined the interplay between access barring and preamble contention. For a single-preamble system, Wu *et al.* [22] developed a fast-adaptive slotted Aloha approach that estimates the number of UEs with active requests (a related estimation approach is presented in [23]). Similar to this paper, a multiple-preamble system is considered by Duan *et al.* [24]. Duan *et al.* examined the total activation time, i.e., the time to complete connection establishment for a given set of UEs, for an unlimited number of transmission attempts. In contrast to the analysis based on complex Markov and combinatorial models in [24], we employ simple elementary steady-state analysis techniques and justify the suitability of these simple analysis techniques for the LTE connection establishment system. An iterative evaluation approach for a detailed, albeit complex Markov queueing model of MTC in LTE RA has been presented by Niyato *et al.* [25].

Generally, analytical modeling of the LTE connection establishment system has so far either focused on rather simple models that capture some of the key mechanisms or quite complex models of relatively high fidelity, albeit at the expense of high complexity. A simple approximate model of the preamble contention was developed in [26]; in contrast to our model, the approximate model [26] does not explicitly consider the backoff nor the limited number of transmission attempts. A simplified model of LTE connection establishment was represented through a G/G/1 queue in [27]. A high-fidelity Markov chain model encompassing physical and medium access control mechanisms of 3GPP Universal Terrestrial Access, which is closely related to LTE, has been developed by Yun [28]. Osti *et al.* [29] developed a Markov model for the queue of the contention resolution messages sent by the eNB to the UEs over the downlink control channel and investigated the impact of control channel limitations. The queueing model assumed a Poisson distribution for the total number of (new and backlogged) UE requests, and Osti *et al.* verified the applicability of the Poisson distribution through simulations with a Poisson process for the generation of new requests. In contrast, we analytically examine the Poisson process characteristics of the number of UE requests participating in the LTE connection establishment and conduct verifying simulations for a range of request generation processes, including non-Poisson processes. Overall, our system model in this paper strives to strike a balance between capturing the key mechanisms of LTE connection establishment with high fidelity, while employing elementary modeling techniques that readily reveal fundamental system dynamics.

## III. Connection Establishment Model

Here, we briefly review the RRC connection establishment, which we refer to in short as "connection establishment," in LTE/LTE-A systems and introduce our model notation. For the distributed user equipment (UE) nodes, we consider two modes of operation, namely idle mode and connected mode. When a UE attached to the home public land mobile network (HPLMN) that does not have some data traffic to send or receive, it normally stays in idle mode. However, if this UE has some data traffic to send to the network or to receive from the network, the mode of operation for this UE needs to transition from idle mode to connected mode. This transition requires two phases of operations, which are the main focus of this paper: Access control [30, Sec. 4.3] (except when Rel-12 UEs are initiating mobile-originating short message service) and RA [8].

We refer to the UEs that need to make this transition as "having a request." Specifically, in our system model, time is

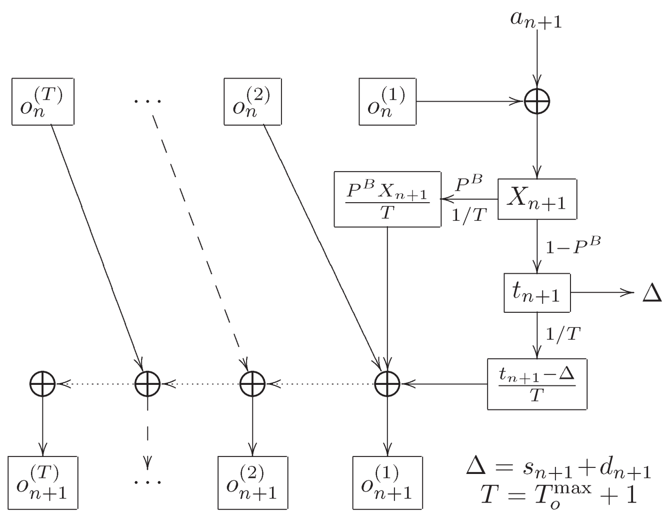


Fig. 1. Model of connection establishment system with a 1BO system. In slot $n+1$, $o_n^{(1)}$ backlogged requests plus $a_{n+1}$ new requests give a total of $X_{n+1}$ UE request candidates. $P^B X_{n+1}$ requests fail the ABC and are uniformly distributed over the next $T_o^{\max}+1$ slots for rejoining the set of UE candidate requests. Similarly, $t_{n+1}-s_{n+1}-d_{n+1}$ requests fail the preamble contention (and remain for retransmission in a future slot) and are uniformly distributed over the next $T_o^{\max}+1$ slots.

slotted, and the (deterministic) integer variable $n$ indexes the successive fixed-duration time slots. As shown in Fig. 1, we denote $a_{n+1}$ for the number of newly generated UE requests (i.e., the number of UEs that initiate the idle to connected mode transition) in a given slot $n+1$. (We mainly consider slot $n+1$ in this initial description as it reduces clutter in the description of the backlog model in Section V-A.) The model notation is summarized in Table I. The $a_{n+1}$ new requests together with $o_n^{(1)}$ old requests that were previously barred or collided in the preamble contention and rescheduled for slot $n+1$ form the set of "candidate UE requests" for slot $n+1$ in our model. We note that the LTE-A standard specifies separate backoff procedures after failing the ABC or the preamble contention, whereby UE requests that failed the preamble contention do not repeat the ABC for retransmissions. For ease of exposition of the backoff process dynamics, we initially consider a simplified model with a single backoff (1BO) system for UE requests failing the ABC or preamble contention, whereby UE requests that failed the preamble contention repeat the ABC for each preamble retransmission. In Section V-C, we present the extension to separate backoff systems for ABC and preamble contention.

Access control in LTE-A has various forms [30, Sec. 4.3]. Up to 3GPP Release-12, all types of UEs are subject to one of the forms of access control, called access class barring [30, Sec 4.3], also known as ABC. Specifically, UEs must go through the ABC when they need to transfer from idle mode to connected mode.

A UE attached to the HPLMN is required to monitor the system information messages broadcast by the central eNB, such as the master information block and system information block (SIB) [31]. SIB2 carries parameters that a UE needs to know in preparation for the ABC and RA. In our model, we consider a static set of parameters and leave the study of dynamic parameter adaptation for future research. The SIB2 contains the

TABLE I
SUMMARY OF MAIN MODEL NOTATIONS

| | |
|---|---|
| **System parameters (constants)** | |
| $P^B$ | Access barring factor (probability): UE request proceeds to preamble contention with prob. $1-P^B$; with prob. $P^B$ the UE is barred (backs off) |
| $O$ | Number of available preambles |
| $T_o^{\max}$ | Maximum backoff time [in slots]: Requests that collide in a slot $n$ are re-transmitted in a (uniformly distr.) slot $n+1,\ n+2,\ \ldots, n+T_o^{\max}+1$ |
| $W$ | Maximum number of preamble transmission attempts |
| **Numbers of newly generated, successful, and dropped UE requests** | |
| $a_{n+1}$ | Number (random var.) of newly generated (arrived) requests in slot $n+1$ |
| $\lambda = \mathbb{E}a_{n+1}$ | Expected number of newly generated requests for a slot |
| $\rho = \lambda/O$ | Normalized load per preamble |
| $s_{n+1}$ | Number (random var.) of successful preamble transmissions (without collision) in slot $n+1$ |
| $\varsigma$ | Steady-state success probability of a preamble transmission in a given slot |
| $d_{n+1}$ | Number of requests dropped at end of slot $n+1$ after having failed in $W$ transmission attempts |
| $\delta$ | Steady-state drop probability of a given request |
| **Total number of transmitted UE requests** | |
| $X_{n+1}$ | Total number (random variable) of candidate requests for transmission in slot $n+1$ |
| $x = \mathbb{E}X_{n+1}$ | Expected total number of candidate requests in a slot |
| $t_{n+1}$ | Total number (random variable) of actually transmitted requests in slot $n+1$ |
| $\theta = \mathbb{E}t_{n+1}$ | Expected (steady-state) value of total number of actually transmitted requests in a slot |
| **Backlog model** | |
| $o_n^{(i)}$ | Total number (random var.) of backlogged (old) requests at end of slot $n$ scheduled for re-joining the set of candidate UE requests in slot $n+i,\ i=1,2,\ldots,T_o^{\max}+1$ |

parameters `ac-BarringFactor` and `ac-BarringTime`, which control the ABC. Specifically, each of the

$$X_{n+1} = a_{n+1} + o_n^{(1)} \tag{1}$$

candidate UE requests independently undergoes the ABC, which we model as follows. With probability $P^B$, the candidate request is barred and has to wait for the access barring time. With the complementary probability $1-P^B$, the UE request proceeds to the RA, i.e., the preamble transmission contention in slot $n+1$.

The access barring time is a uniformly randomly distributed time between prescribed minimum and maximum values. The access barring times in the LTE/LTE-A standard (except for the zero access barring time option) are longer than the backoff times after a preamble collision. However, to avoid unnecessary clutter in our analysis, we consider the same random waiting times for the access barring and the backoff after a preamble collision in our model.

After clearing the ABC, a UE moves on to the contention-based RA procedure, which we model as follows. The UE uniformly randomly selects a preamble from a set of $O$ preambles available for RA and transmits the request using the chosen preamble. If a given UE request is the only one being transmitted using a given preamble in a given slot $n+1$, then this UE request is considered successful and contributes to the number $s_{n+1}$ of successful UE requests in slot $n+1$. On the other hand, if more than one UE request is transmitted using the same preamble in a given slot, then we consider a collision to have occurred on that preamble. Although, it may be possible to retrieve some request information in this case, we model a worst-case scenario and consider all requests that used the particular preamble as irrecoverable.

If a UE request conducted its $W$th preamble transmission attempt in a given slot $n+1$ and suffered a collision in this $W$th attempt, then the request drops from the connection establishment model and contributes to the number $d_{n+1}$ of dropped UE requests in slot $n+1$. If a collided UE request has undergone less than $W$ preamble transmission attempts, then the UE backs off according to a backoff interval parameter received from the eNB via a medium-access-control protocol data unit message. Specifically, the UE backs off by uniformly randomly selecting a timeout from the backoff interval $[0,\ T_o^{\max}]$ slots, as examined in detail in Section V-A. After the timeout expires, a backlogged UE request rejoins the set of candidate UE requests in our 1BO model and again undergoes the ABC and, if clearing ABC, retransmits the request using another independently randomly chosen preamble.

## IV. Connection Establishment Analysis

We initially consider a single-preamble ($O=1$) system in Section IV-A and subsequently map the multiple-preamble ($O>1$) system to $O$ single-preamble systems in Section IV-B. Our analytical strategy here is to consider steady-state Poisson distributions for key random variables characterizing the connection establishment system. In Section V we will justify the steady-state Poisson distributions through a detailed analysis of the uniform backoff over a maximum of $T_o^{\max}$ slots.

### A. Single-Preamble $O=1$ Contention

*1) Equilibrium Condition:* The connection establishment system modeled in Fig. 1 is neither a source nor a sink for UE requests. That is, the arrivals of newly generated UE requests and departures of (either successful or dropped) UE requests must balance. More specifically, the long-run expected number $\mathbb{E}a_{n+1}$ of UEs with newly generated requests in a slot must be in equilibrium with the sum of the long-run expected number $\mathbb{E}s_{n+1}$ of successful UE requests in a slot and the long-run expected number $\mathbb{E}d_{n+1}$ of UE requests dropped at the end of the slot, i.e.,

$$\mathbb{E}a_{n+1} = \mathbb{E}s_{n+1} + \mathbb{E}d_{n+1}. \tag{2}$$

*2) Expected Number $\mathbb{E}a_{n+1}$ of new UE Requests:* We initially consider a Poisson arrival process with rate $\lambda$ (requests/slot) for the newly generated UE requests. For this Poisson request arrival model, there are an expected number of $\lambda$ new requests per slot, i.e.,

$$\mathbb{E}a_{n+1} = \lambda. \tag{3}$$

*3) Expected Number $\mathbb{E}s_{n+1}$ of Successful UE Requests:* We proceed to derive expressions for the expected number $\mathbb{E}s_{n+1}$ of successful preamble transmissions in a slot here (and the expected number $\mathbb{E}d_{n+1}$ of dropped requests at the end of a slot in Section IV-A4). We derive the expressions for $\mathbb{E}s_{n+1}$ and $\mathbb{E}d_{n+1}$ in terms of the expected number $\mathbb{E}X_{n+1}$ of UE request candidates in slot $n+1$. Both newly generated requests and old requests that have previously been barred or collided in the preamble contention (and have not yet exhausted their $W$ transmission attempts) contribute to $\mathbb{E}X_{n+1}$.

A key step in our derivation is to model the stochastic process formed by the sequence of random variables $X_{n+1}$ representing the number of UE request candidates in the successive slots $n, n+1, n+2, \ldots$, to have the characteristics of a Poisson process. In Section V-B, we will examine the stochastic process $X_{n+1}$ and demonstrate that the Poisson process model is appropriate for the process $X_{n+1}$. For now, we denote $x$ for the steady-state expectation of $X_{n+1}$, i.e.,

$$x = \mathbb{E}X_{n+1} \tag{4}$$

and proceed to derive the expectations $\mathbb{E}s_{n+1}$ and $\mathbb{E}d_{n+1}$.

With $X_{n+1}$ UE request candidates and a barring probability of $P^B$, there are

$$t_{n+1} = X_{n+1}(1-P^B) \tag{5}$$

UE requests that clear barring and are actually transmitted in slot $n+1$. Thus, the expected number of actually transmitted UE requests on the considered single preamble ($O=1$) in a slot is

$$\mathbb{E}t_{n+1} = x(1-P^B) =: \theta. \tag{6}$$

For contention with $O=1$ preamble, a given slot $n+1$ has one successful UE request transmission if exactly one request is transmitted in the slot. For a Poisson process model of $X_{n+1}$, the number $t_{n+1}$ of actually transmitted UE requests in the slot has a Poisson distribution with parameter (mean) $\theta$ [see (6)]. A Poisson distributed random variable with mean $\theta$ attains the value one with probability $\theta e^{-\theta}$ [32]. Thus,

$$\mathbb{E}s_{n+1} = \theta e^{-\theta} \tag{7}$$

gives the expected number successful UE request transmissions per slot. Normalizing the expected number of successful UE request transmissions $\mathbb{E}s_{n+1} = \theta e^{-\theta}$ by the expected number of actual UE transmissions $\mathbb{E}t_{n+1} = \theta$ in a slot gives the steady-state probability of a given actual UE transmission being successful in a given slot as

$$\varsigma = e^{-\theta}. \tag{8}$$

*4) Expected Number $\mathbb{E}d_{n+1}$ of Dropped UE Requests:* If a UE request is unsuccessful (collides) in its $W$th transmission attempt, it is dropped at the end of the slot of the $W$th attempt. We model subsequent transmission attempts to have negligible correlation, i.e., to be approximately independent. For the model of independent transmission attempts, the steady-state drop probability of a given UE request failing (colliding) in all of its $W$ attempts is

$$\delta = (1-\varsigma)^W. \tag{9}$$

In steady state, an expected number of $\mathbb{E}a_{n+1} = \lambda$ new requests enter the connection establishment system per slot.

Each of these new requests fails in steady state with probability $\delta$ in all of its $W$ transmission attempts. Thus, the expected number of UE requests dropping per slot from the connection establishment system in steady state is

$$\mathbb{E}d_n = \lambda\delta. \tag{10}$$

*5) Equilibrium Condition:* Substituting the results from Sections IV-A2 through IV-A4 into (2) gives the equilibrium condition in terms of the expected number $\theta$ of transmitted UE requests in a given slot, i.e.,

$$\frac{\theta}{\lambda} = \frac{1 - \left(1 - \mathrm{e}^{-\theta}\right)^W}{\mathrm{e}^{-\theta}} \tag{11}$$

which can be readily solved numerically for $\theta$. From (6) we obtain the corresponding expected number of UE request candidates $x = \theta/(1 - P^B)$.

### B. Multipreamble $O > 1$ Contention

We now extend the model for single-preamble $O = 1$ contention to the multipreamble $O > 1$ case. Our multiple-preamble model critically relies on the Poisson process model for the number $a_{n+1}$ of newly generated UE requests per slot (see Section IV-A2) and the Poisson process model for the number $X_{n+1}$ of UE request candidates in a slot (see Section IV-A3). Poisson processes satisfy the "Poisson splitting" property [32], i.e., independently randomly splitting off events from a Poisson process results in a new Poisson process. In the multipreamble contention, each actually transmitted UE request independently and uniformly and randomly selects one of the $O$ preambles with probability $1/O$ for transmission.

We split the overall model for the RA system with $O, O > 1$, preambles into $O$ independent "per-preamble" models, one model for each of the $O$ preambles. Each of the $O$ independent "per-preamble" models has identical distributions for the respective random variables. Each per-preamble model pertains primarily to the participation of an independently and randomly selected subset of $1/O$ of the total number of UE candidate requests in the preamble contention on a given preamble out of the $O, O > 1$, orthogonal preambles. Collided preamble transmissions join the overall backoff model, i.e., there is *one* backoff model for the entire connection establishment system with $O, O > 1$, preambles, as presented in Section V-A.

We proceed to examine one of the $O$ "per-preamble" models in detail and identify its differences with respect to the $O = 1$ model presented in Section IV-A. New UE requests are overall generated at rate $\lambda$ (requests/slot). Each request will eventually be transmitted on one of the $O, O > 1$, preambles. Thus, the arrival rate of new UE requests to a given per-preamble model is effectively as follows:

$$\mathbb{E}a_{n+1} = \frac{\lambda}{O} =: \rho. \tag{12}$$

Similarly, each of the actual UE request transmissions selects one of the $O$ preambles, resulting in an expected number of

$$\mathbb{E}t_{n+1} = \frac{x(1 - P^B)}{O} =: \theta \tag{13}$$

transmitted UE requests per preamble per slot. (Notice that for $O = 1$, (13) simplifies to (6) of the single-preamble contention). The "per-preamble" expected number $\theta$ of transmitted UE requests defined in (13) can be analogously employed as the single-preamble $\theta$ defined in (6) for the evaluation of the expected numbers of successful and dropped UE request in Sections IV-A3 and IV-A4.

Similar to the analysis in Section IV-A5, we obtain thus an equilibrium condition analogous to (11), but with $\lambda$ in (11) replaced by the per-preamble load $\rho$ (12) and with $\theta = x(1 - P^B)/O$ as defined in (13). This new form of the equilibrium condition can readily be numerically solved to obtain the expected total number $x$ of candidate UE requests in the overall RA system with $O, O > 1$, preambles.

### C. Delay and Throughput

From the solution $\theta$ to the equilibrium condition (11), we obtain the success probability $\varsigma$ of a given preamble transmission attempt through (8) and the drop probability $\delta$ of a given UE request through (9). The steady-state throughput of successful UE requests completing the connection establishment procedure per slot is then

$$TH = \lambda(1 - \delta). \tag{14}$$

There are two main components for evaluating the mean delay $D$ in slots for the completion of the connection establishment procedure by a successful request. First, the delay component $D_c$ accounts for the delays incurred due to the possible number of $c, c = 0, 1, \ldots, W - 1$, collisions in the preamble contention. Each collision in the preamble contention is preceded by an expected number of $P^B/(1 - P^B)$ failures in the access barring (following the properties of the geometric distribution with success probability $P^B$ [32]). Each failure in access barring requires a backoff with a mean delay of $1 + T_o^{\max}/2$ slots. The preamble collision necessitates one additional backoff with mean delay $1 + T_o^{\max}/2$ slots. Thus, the UE experiences an expected total number of $1/(1 - P^B)$ backoffs for each experienced collision. The expected number of collisions is

$$\mathbb{E}c = \sum_{c=0}^{W-1} c \cdot \frac{(1-\varsigma)^c \varsigma}{\sum_{k=0}^{W-1}(1-\varsigma)^k \varsigma} \tag{15}$$

$$= \frac{(1-\varsigma)\left[1 + (W-1)\delta\right] - W\delta}{\varsigma(1-\delta)} \tag{16}$$

whereby (16) follows through algebraic simplifications from (15). Thus, $D_c = (1 + T_o^{\max}/2)\mathbb{E}c/(1 - P^B)$.

Second, after suffering a mean number $\mathbb{E}c$ of collisions, a successful UE request undergoes once more an expected number of $P^B/(1 - P^B)$ failures in the access barring, followed by a successful preamble transmission, i.e.,

$$D_s = \left(1 + \frac{T_o^{\max}}{2}\right)\frac{P^B}{1 - P^B}. \tag{17}$$

The overall expected delay in slots is thus

$$D = \left(1 + \frac{T_o^{\max}}{2}\right) \frac{P^B + \mathbb{E}c}{1 - P^B}. \tag{18}$$

## V. Uniform Backoff over a Maximum of $T_o^{\max}$ Slots

Here, we examine the impact of backoff of barred or collided UE requests over the backoff interval of $[0, T_o^{\max}]$ slots, i.e., the barred/collided requests are rescheduled for one of the subsequent $T_o^{\max} + 1$ slots. We examine the random variable representing the number of candidate UE requests $X_{n+1}$. We will demonstrate that, for connection establishment systems with high barring probability $P^B$ and long maximum timeout $T_o^{\max}$, the stochastic process $X_{n+1}$ tends to a Poisson process.

### A. *Backlog Model*

As shown in Fig. 1, the $P^B X_{n+1}$ barred UE requests and the $t_{n+1} - s_{n+1} - d_{n+1}$ UE requests that collided in the preamble contention in slot $n+1$ and have not yet exhausted their $W$ transmission attempts (i.e., are remaining for retransmission(s) after transmission in slot $n+1$) enter the backoff. With uniform backoff, they are uniformly randomly scheduled for rejoining the set of candidate UE requests in one of the subsequent $T_o^{\max} + 1$ slots.

We model the backlogged (old) UE requests through backlog registers $o_{n+1}^{(1)}, o_{n+1}^{(2)}, \ldots, o_{n+1}^{(T_o^{\max}+1)}$ (see Fig. 1). We define $o_{n+1}^{(r)}$ to be a random variable denoting the number of UE requests that, by the end of slot $n+1$, have been scheduled for rejoining the candidate UE request set in slot $n+1+r$, i.e., $r$ slots ahead of the present slot index $n+1$. Specifically, $o_{n+1}^{(T_o^{\max}+1)}$ (in the bottom left corner of Fig. 1) denotes the number of UE requests that, at the end of slot $n+1$, have been scheduled for rejoining in slot $n + T_o^{\max} + 2$. Note that, at the end of slot $n+1$ only UE requests that were barred or collided in slot $n+1$ contribute toward $o_{n+1}^{(T_o^{\max}+1)}$, as slot $n + T_o^{\max} + 2$ corresponds to the maximum possible backoff duration $T_o^{\max} + 1$, i.e.,

$$o_{n+1}^{(T_o^{\max}+1)} = \frac{P^B X_{n+1} + (1 - P^B) X_{n+1} - s_{n+1} - d_{n+1}}{T_o^{\max} + 1} \tag{19}$$

$$= \frac{a_{n+1} + o_n^{(1)} - s_{n+1} - d_{n+1}}{T_o^{\max} + 1}. \tag{20}$$

The other backlog registers for slot $n+1$, i.e., the registers $o_{n+1}^{(r)},\ r = 1, \ldots, T_o^{\max}$, receive contributions from the backlog registers at the end of the preceding slot $n$ and from the barred/collided requests in slot $n+1$. For example, the backlogged UE requests in $o_n^{(T_o^{\max}+1)}$ (in the top left corner of Fig. 1) are copied over to $o_{n+1}^{(T_o^{\max})}$. In addition, the barred/collided UE requests from slot $n+1$ that are scheduled for rejoining in slot $n + 1 + T_o^{\max}$ contribute to the backlog register $o_{n+1}^{(T_o^{\max})}$ at the end of slot $n+1$. That is, with each new slot, the backlogged UE requests move effectively one slot closer to their retransmission slot and may receive a contribution from the barred/collided UEs in the new slot. Formally, for $r = 1, 2, \ldots, T_o^{\max}$

$$o_{n+1}^{(r)} = o_n^{(r+1)} + \frac{a_{n+1} + o_n^{(1)} - s_{n+1} - d_{n+1}}{T_o^{\max} + 1}. \tag{21}$$

The introduced backlog register model applies to both systems with $O = 1$ preamble and systems with $O,\ O > 1$, preambles. In either case, the UE requests that collided in slot $n+1$ on the $O = 1$ preamble or on the $O,\ O > 1$, preambles are collected in *one* common set of backlog registers $o_{n+1}^{(1)}, o_{n+1}^{(2)}, \ldots, o_{n+1}^{(T_o^{\max}+1)}$. When UE requests from a given backlog register rejoin the set of candidate UE requests in a system with $O,\ O > 1$, preambles, they are independently, uniformly randomly assigned to one of the $O$ per-preamble models introduced in Section IV-B. That is, a given UE request joins a given per-preamble model only for a given transmission attempt, and may join any of the $O,\ O > 1$, per-preamble models for the next transmission attempt. The $O,\ O > 1$, per-preamble models are thus stochastically independent across multiple transmission attempts.

### B. *Poisson Model Approximation for $o_n^{(1)}$ and $X_{n+1}$*

We now examine the backlog model from Section V-A more closely and develop an approximation for the distribution of the random variable $o_n^{(1)}$ representing the number of backlogged UE requests rejoining the set of UE candidate requests in slot $n+1$ and the random variable $X_{n+1}$ representing the number of UE candidate requests in slot $n+1$. The key approximation modeling step is to consider a limiting case of the connection establishment system in which no UE requests enter or leave the system but rather repeatedly circulate through the system. We formally show in this section that, for this circulatory system, the processes $o_n^{(1)}$ and $X_{n+1}$ tend to Poisson processes for long timeouts $T_o^{\max} \to \infty$. In Section VI, we numerically examine how closely the equilibrium analysis from Section IV, which was based on Poisson processes, characterizes practical connection establishment systems.

*Definition 1:* We define the "circulatory connection establishment system" as a connection establishment system with some initial numbers of UE requests in the backlog registers $o_n^{(r)}, r = 1, \ldots, T_o^{\max} + 1$, but without new UE request arrivals and without UE request departures.

In the illustration in Fig. 1, the circulatory connection establishment system corresponds to a system with the arrows for arrivals $a_{n+1}$ and departures $s_{n+1} + d_{n+1}$ removed.

For the circulatory connection establishment system, the model from Section V-A simplifies as follows. The number $a_{n+1} + o_n^{(1)} - s_{n+1} - d_{n+1}$ of barred or collided UE requests in (21) becomes equal to the number $o_n^{(1)}$ of backlogged requests rejoining the set of UE candidate requests in slot $n+1$. Each of the $o_n^{(1)}$ requests contributes to the backlog register $o_{n+1}^{(r)}$ with probability $1/(1 + T_o^{\max})$, which we model through a family $(U_n(k)),\ \ k = 0, 1, \ldots, o_n^{(1)}$ of independent and identically distributed uniform random variables on $\{1, \ldots, T_o^{\max} + 1\}$. Thus, (21) and (20) simplify to

$$o_{n+1}^{(r)} = o_n^{(r+1)} + \sum_{k=1}^{o_n^{(1)}} 1_{\{U_n(k)=r\}}, r = 1, \ldots, T_o^{\max} \tag{22}$$

$$o_{n+1}^{(T_o^{\max}+1)} = \sum_{k=1}^{o_n^{(1)}} 1_{\{U_n(k)=T_o^{\max}+1\}}. \tag{23}$$

We define the total number of UE requests circulating as

$$N := \sum_{r=1}^{T_o^{\max}+1} o_n^{(r)} \tag{24}$$

which is a constant with respect to the slot index $n$.

The random variables $(o_n^{(r)})_{r=1,\ldots,T_o^{\max}+1}$ in the defined circulatory RA system are not independent, neither for finite $T_o^{\max}$ nor for $T_o^{\max} \to \infty$. Nevertheless, we show in Appendix A for the random variables $o_n^{(r)}$:

*Proposition 1:* For a given maximum timeout $T_o^{\max}$, the distribution of the number $o_n^{(r)}$ of UE requests in backlog register $r$, $r = 1, \ldots, T_o^{\max} + 1$, in the circulatory connection establishment system converges over a long time horizon ($n \to \infty$) to the stationary binomial distribution with parameters $(2(T_o^{\max} - r + 2))/((T_o^{\max} + 1)(T_o^{\max} + 2))$ and $N$, i.e.,

$$o_n^{(r)} \sim BIN\left(\frac{2(T_o^{\max} - r + 2)}{(T_o^{\max} + 1)(T_o^{\max} + 2)}, N\right). \tag{25}$$

Intuitively, the backlog registers "fill up linearly" according to the distribution (25) as the number of slots $r$ until rejoining the set of UE candidate requests decrements from $T_o^{\max}$ to 1. For instance, the backlog register at the mid-point $r = T_o^{\max}/2$ holds on average roughly $N/T_o^{\max}$ UE requests, i.e., it is "half as full" as the backlog register $r = 1$ that feeds its UE requests into the set of candidate UE requests.

We proceed to examine the random variable $o_n^{(1)}$ representing the number of UE requests rejoining in slot $n + 1$. For long maximum timeout ($T_o^{\max} \to \infty$) with

$$\frac{N}{T_o^{\max}} \to \omega \tag{26}$$

the stationary binomial distribution (25) converges to a limiting Poisson distribution with the following parameter:

$$\lim_{T_o^{\max}\to\infty} \frac{2N(T_o^{\max} + 1)}{(T_o^{\max} + 1)(T_o^{\max} + 2)} = 2\omega \tag{27}$$

i.e.,

$$o_n^{(1)} \sim POI(2\omega). \tag{28}$$

Moreover, the random variables $o_n^{(1)}$ in successive slots $n, n + 1, \ldots$ are asymptotically independent, as examined in Appendix B.

In the circulatory connection establishment system, the number $X_{n+1}$ of UE candidate requests is equal to the number $o_n^{(1)}$ of backlogged UE requests rejoining the connection establishment. We have thus established that the random variables $o_n^{(1)}$ and $X_{n+1}$ in the circulatory connection establishment system with large timeout ($T_o^{\max} \to \infty$) become independent and identically distributed random variables, specifically Poisson random variables with mean $2\omega$.

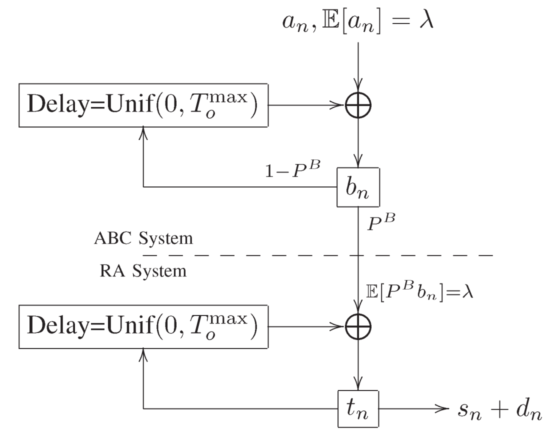


Fig. 2. Two backoff (2BO) system: New UE requests pass through the ABC system and then proceed to the RA system for preamble contention.

*Theorem 1:* For the circulatory LTE/LTE-A connection establishment system with large timeout ($T_o^{\max} \to \infty$), the process of the number $X_{n+1}$ of candidate UE requests in a slot tends to become equivalent to a Poisson process with rate $2\omega$ (requests/slot).

For the circulatory connection establishment system, the mean number $2\omega = 2N/T_o^{\max}$ of UE candidate requests is equivalent to the mean number $x$ of candidate requests in the more general system considered in Section IV. For the more general system in Section IV, we obtained $x$ through the solution of the equilibrium condition (11). Practical connection establishment systems can approximate the circulatory system with $T_o^{\max} \to \infty$, e.g., through setting a high barring probability $P^B$ that repeatedly recirculates a high portion of the UE request through the backoff and setting a long $T_o^{\max}$. For such settings, Theorem 1 justifies the approximate modeling of the number of UE request candidates in a slot with a stationary Poisson random variable in Section IV-A3.

### C. Extension to Separate Backoff Systems

We now extend the model from Section III with a 1BO system accommodating both UEs that failed the ABC and the RA preamble contention (RA) to the model with two separate backoff (2BO) systems for ABC and RA, as shown in Fig. 2. In the steady state, $\mathbb{E}[a_n] = \lambda$ new UE requests enter the ABC system per slot, whereas the same expected number of $P^B\mathbb{E}[b_n] = \lambda$ UE requests (some new, some previously backlogged in the ABC system) exit the ABC system to enter the RA system. Modeling the number of UE requests transmitted in a slot $t_n$ as a Poisson random variable with mean $\mathbb{E}[t_n] = \theta$, the equilibrium of the RA system (and overall 2BO system) is characterized with exactly the condition given by (11). Hence, the 2BO system results in exactly the same expected number $\theta$ of UE requests transmitted (pre preamble) per slot as the 1BO system, as well as the same success probability as per (8) and drop probability (9) and throughput (14). The mean delay in the ABC system is $D_{\mathrm{ABC}} = [P^B/(1-P^B)](1+T_o^{\max}/2)$, and the mean delay in the RA system is $D_{\mathrm{RA}} = E_c(1+T_o^{\max}/2)$, with $E_c$ from (16), giving the total mean delay for the 2BO model as $D = D_{\mathrm{ABC}} + D_{\mathrm{RA}}$. The analysis of the uniform backoff in Section V-B applies analogously to the ABC system, i.e., the UE request process entering the RA system becomes equivalent to a Poisson process for a circulatory ABC system with long $T_o^{\max}$.

## VI. Numerical Analysis

In this section, we examine the connection establishment procedure through numerical evaluation based on the equilibrium analysis in Section IV and simulations. We use simulation models developed with C++. For each simulation scenario, we conduct multiple independent replications to achieve 95% confidence intervals with less than 5% relative error or complete at least one million simulated slots. We compare key performance metrics of the connection establishment, namely success probability $\varsigma$, drop probability $\delta$, mean throughput $TH$, and mean delay $D$ obtained from the equilibrium analytical model in Section IV with corresponding simulations. We plot these metrics as a function of normalized load, i.e., the mean number of new UE requests per preamble per slot, $\rho = \lambda/O$. In the plots, we denote "Ana" for the curves obtained from the analysis model, whereas simulation points for various parameters are marked by the parameter values. We set the number of preambles to $O = 54$ and the number of transmission attempts to $W = 4$ throughout this section.

LTE-A systems with MTC devices may receive nonsynchronized (uncorrelated) requests or requests that are highly synchronized (correlated), e.g., when a large number of machines try to simultaneously connect in a burst, e.g., after a power outage, or when many sensors respond to a common event [33]–[36]. We have conducted evaluations for a Poisson arrival process modeling uncorrelated requests and Bernoulli process and Markov-modulated Poisson process (MMPP) arrivals modeling correlated arrivals. For the Poisson arrival process, the number $a_{n+1}$ of newly generated UE requests follows a stationary independent Poisson distribution with mean (and variance) $\lambda$ (requests/slot). For the Bernoulli arrival process, the successive numbers $a_{n+1}$ of new UE requests are independent Bernoulli distributed random variables taking on the values 0 and $2\lambda$ with probability 0.5, which results in the maximum variance $\lambda^2$ of a Bernoulli random variable [37], [38]. The Bernoulli arrivals model request synchronization in that either zero or $2\lambda$ new request are generated in a given slot. For the MMPP arrival process, we consider a low-request-rate state with $\lambda/h$ (requests/slot) and a high-request-rate state with $h\lambda$ (requests/slot) with mean sojourn times of 100 slots and $100/h$ slots, respectively. The parameter $h$ represents the ratio of the high request rate to the mean request rate $\lambda$ and is initially set to $h = 5$. The MMPP is a highly challenging request generation process since, in contrast to Poisson or Bernoulli arrivals, the distribution of the numbers $a_{n+1}$ of new UE requests is not unimodal. The MMPP models request synchronization in that

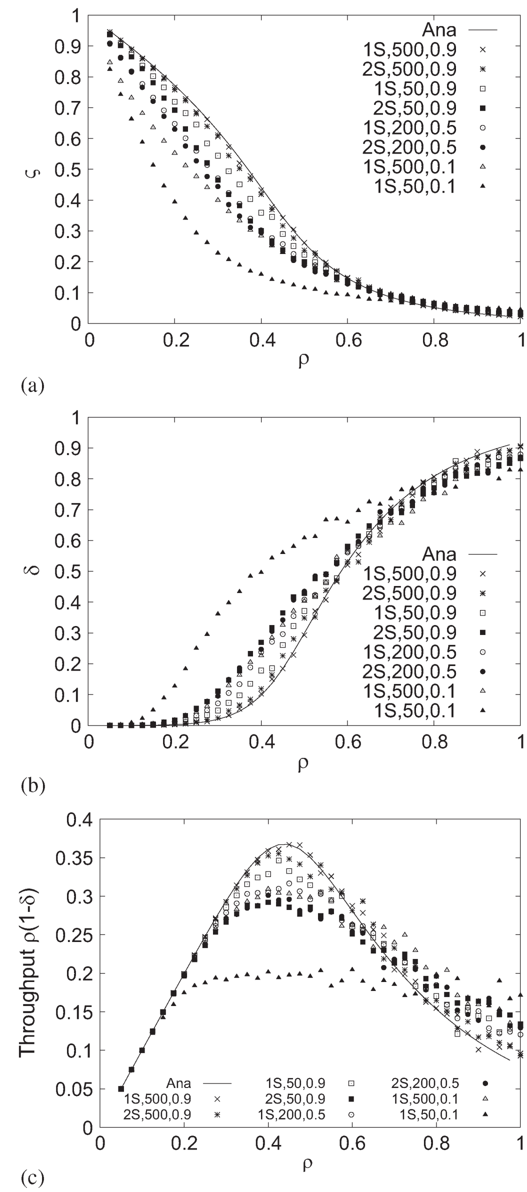


Fig. 3. Connection establishment performance for MMPP arrivals for different combinations of backoff system (1BO or 2BO), timeout $T_o^{\max}$, and barring probability $P^B$ (e.g., "1S, 500, 0.9" means "1BO system simulation, $T_o^{\max} = 500$, $P^B = 0.9$") as a function of normalized load $\rho = \lambda/O$. (a) Success probability $\varsigma$, (b) Drop probability $\delta$, (c) Throughput.

the rate of new requests either persist at a low Poisson rate or a high Poisson rate for the sojourn times in the respective states. Due to space constraints, we focus in the following mainly on the challenging MMPP model with persistently low or high request arrival rates and summarize the main results for Poisson and Bernoulli arrivals.

### A. Impact of Timeout $T_o^{\max}$

Focusing initially on the 1BO system, we observe from Fig. 3 that for the two-state MMPP arrivals, the discrepancies between analysis and simulations are quite pronounced for short timeout $T_o^{\max} = 50$ in conjunction with low barring probability $P^B = 0.1$. Long timeouts with $T_o^{\max} = 500$ reduce the discrepancies in the mid-range of Fig. 3(c) to roughly a third of the discrepancy for $T_o^{\max} = 50$. These results indicate that long maximum timeouts $T_o^{\max}$ are quite effective in uniformly redistributing arrivals from a two-state MMPP so that the system dynamics approach those of a Poisson system model. Effectively, the independent uniform random distribution of the barred/collided UE requests over $T_o^{\max} + 1$ future slots acts to reduce the correlation among the numbers of UEs participating in the connection establishment procedure in the subsequent slots. For sufficiently large $T_o^{\max}$, the numbers of participating UE requests in the successive slots become independent and identically distributed Poisson random variables, i.e., constitute a Poisson process. In other words, uniform backoff over long timeouts sufficiently redistributes (spreads out) the barred or collided UE requests so that the resulting simulated (real) system performance is closely approximated by the analytical model based on the limiting ($T_o^{\max} \to \infty$) Poisson process in Theorem 1.

This redistribution effect of the timeout is further corroborated for the Bernoulli arrivals considered in Fig. 5. Specifically, we observe from Fig. 5(a) for the 2BO system with $P^B = 0.1$ that the simulated throughput with $T_o^{\max} = 10$ (2S, 10, 0.1 curve) is much closer to the analytical Poisson model than the corresponding simulated throughput with $T_o^{\max} = 0$ (2S, 0, 0.1 curve). With $T_o^{\max} = 0$, barred or collided requests immediately rejoin in the next slot. Comparing Figs. 3 and 5, we observe that the curves for the simulated Bernoulli arrivals approach the analytical Poisson model curve for much shorter timeouts $T_o^{\max}$ compared with the curves for the simulated MMPP arrivals. This is mainly because the unimodal Bernoulli arrival model does not have the persistent phases of low or high requests of the MMPP model; instead, Bernoulli requests are only synchronized for a given slot, i.e., either 0 or $2\lambda$ new UE requests arrive in a slot. In additional evaluations, we found that for simulated Poisson request arrivals, very short timeouts of $T_o^{\max} = 1$ are sufficient to very closely approximate (so that the curves essentially overlap) the analytical Poisson model for small barring probability $P^B = 0.1$.

### B. Impact of Barring Probability $P^B$

We observe from Fig. 3 that, for the 1BO system, with MMPP arrivals, the high barring probability $P^B = 0.9$ leads already for short timeout $T_o^{\max} = 50$ to a relatively close approximation of the simulation results to the Poisson process analysis; whereas for a long timeout $T_o^{\max} = 500$, there is very close agreement between the simulated system performance for two-state MMPP arrivals and the analytical model based on Poisson random variables. Fig. 5 indicates that Bernoulli arrivals, with $P^B = 0.9$ and a timeout of $T_o^{\max} = 0$, i.e., immediate rejoining of barred or collided requests in the next slot, result in close approximation of the analytical model. The $P^B = 0.9$ barring probability circulates UE requests on average $P^B(1 - P^B)$ times through the backoff before letting a UE request proceed to the preamble transmission. With each circulation through the backoff, a request is uniformly randomly distributed over $T_o^{\max} + 1$ slots. This repetitive random redistribution effectively smoothes the two-state MMPP and Bernoulli arrivals so that the success and drop probabilities and the throughput

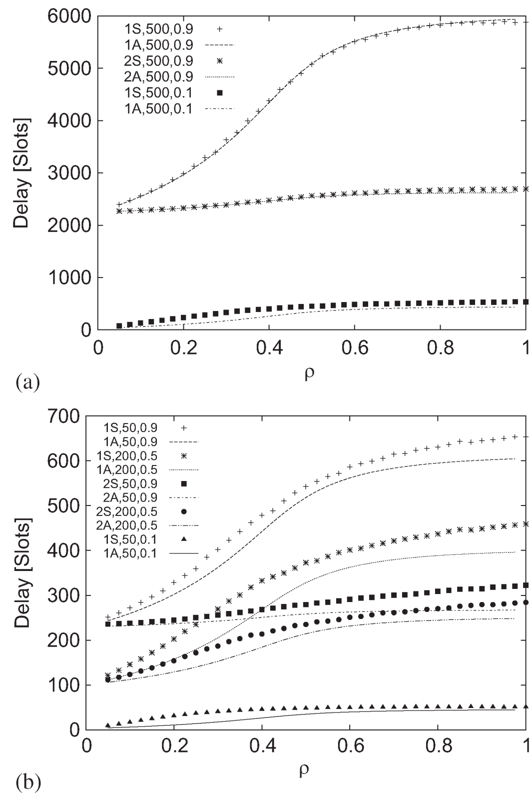


Fig. 4. Mean delay for MMPP arrivals for combinations of backoff system (1BO or 2BO), timeout $T_o^{\max}$, and barring probability $P^B$. (a) Mean Delay for $T_o^{\max} = 500$ (b) Mean Delay, $T_o^{\max} = 50, 200$.

of successful connection establishments closely approximate the corresponding metrics with Poisson request arrivals. For a per-preamble MMPP traffic load of $\rho = 0.3$ and $T_o^{\max} = 50$, for instance, the drop probability in Fig. 3(c) is reduced from approximately $\delta = 0.36$ for the low barring probability $P^B = 0.1$ to around 0.05 for $P^B = 0.9$. As observed in Fig. 4(b), these improvements come at the expense of roughly tenfold increased delay due to the repetitive recirculation.

We observe from Figs. 3 and 5 that simulation results for the 2BO system for both MMPP and Bernoulli arrivals exhibit slightly larger discrepancies from the analytical Poisson model than the 1BO system. However, Figs. 4 and 5 indicate that the 2BO system has substantially lower delays. In the 1BO system, a UE request undergoes ABC for each preamble transmission attempt; whereas, in the 2BO system, a UE request undergoes ABC only once. Thus, in the 2BO system, the UE request traffic is smoothed only once by the (on average $P^B(1 - P^B)$) circulations through the ABC backoff system and experiences then one additional circulation through the RA backoff system for each failed preamble transmission attempt. In contrast, in the 1BO system, a UE request undergoes the (on average $P^B(1 - P^B)$) circulations through the backoff system for each preamble transmission attempt.

We conclude by observing from Figs. 3 and 4 that a moderately high access barring probability, e.g., $P^B = 0.5$, in conjunction with a moderately long timeout, e.g., $T_o^{\max} = 200$, results for the challenging two-state MMPP arrivals in throughput levels reasonably close to the levels for Poisson arrivals.

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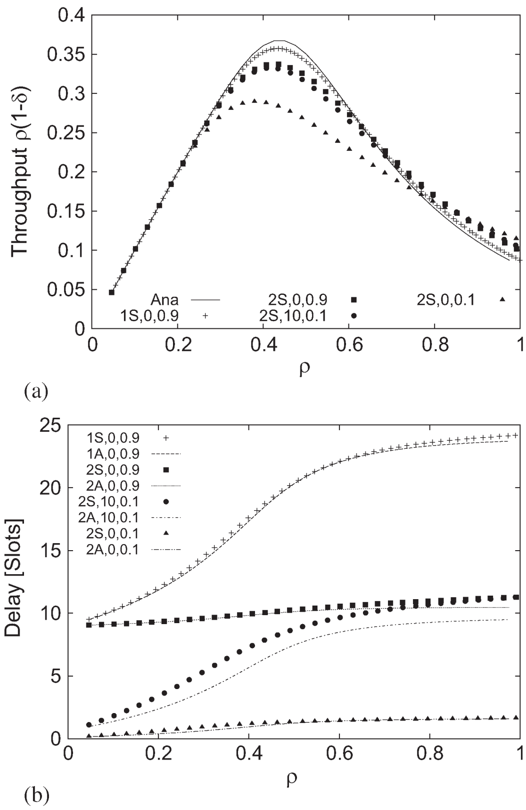


Fig. 5. Throughput and mean delay for Bernoulli arrivals for combinations of backoff system (1BO or 2BO), timeout $T_o^{\max}$, and barring probability $P^B$. (a) Throughput (b) Mean Delay.

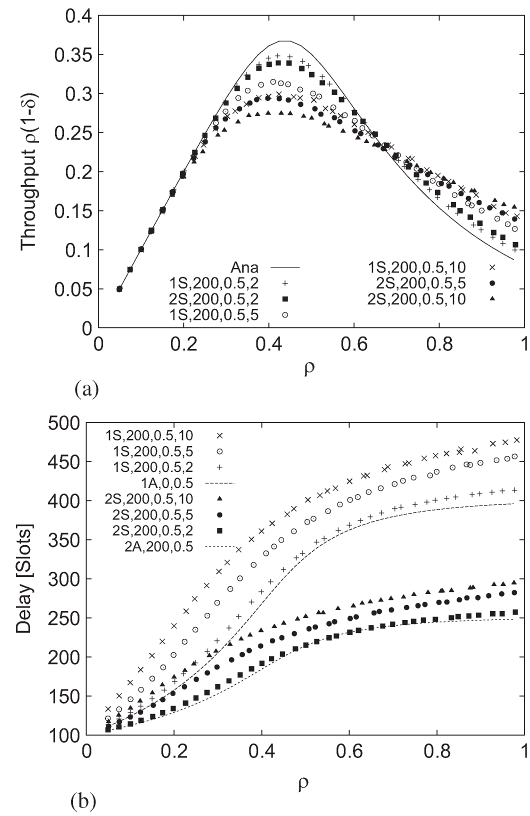


Fig. 6. Throughput and mean delay for MMPP arrivals for combinations of backoff system (1BO or 2BO) and ratio of high to mean arrival rates ($h = 2$, 5, or 10). Fixed parameters: timeout $T_o^{\max} = 200$ and barring probability $P^B = 0.5$. (a) Throughput (b) Mean Delay.

This relatively good throughput for the challenging 2MMPP arrival process is achieved at the expense of moderate delays below 500 slots.

### C. Impact of Request Traffic Characteristics

For the moderately high barring probability $P^B = 0.5$ and timeout $T_o^{\max} = 200$, we proceed to examine the impact of the high-to-mean request rate ratio $h$ of the MMPP request traffic. So far, we had considered a high-to-mean request rate ratio of $h = 5$, which corresponds to a high-to-low request rate ratio of $h^2 = 25$. A higher high-to-mean request rate ratio $h$ models a higher level of request synchronization, i.e., the phases (bursts) of high request rates are more intense and shorter. We observe from Fig. 6 that highly bursty requests with a high request rate $h = 10$ times above the mean rate result in larger discrepancies than the high-to-mean request rate ratio of $h = 5$ considered so far. For the highly bursty requests with $h = 10$, the high request rate is $h^2 = 100$ times the low request rate, i.e., the high-to-low request rate ratio for $h = 10$ is four times the high-to-low request rate ratio with $h = 5$. Nevertheless, the deviations between the simulated throughput and delay and the analytical throughput and delay for $h = 10$ are less than about 1.75 times the corresponding deviations for $h = 5$. We also observe from Fig. 6 that, for a low degree of request synchronization (burstiness) with a high-to-mean request rate ratio of $h = 2$, the simulated curves quite closely approach the analytical model.

## VII. Conclusion

We have derived equilibrium models for the connection establishment system in LTE/LTE-A networks. Our equilibrium approach models both the ABC and RA preamble contention through Poisson random variables. Our analysis is applicable to both network operation with a single preamble and multiple preambles. We have formally shown that the uniform backoff in LTE/LTE-A connection establishment results in stationary Poisson distributions for key system variables.

We have verified that, for two-state Markov Modulated Poisson Process UE request arrivals the uniform backoff in LTE/LTE-A connection establishment can lead to system dynamics that closely approximate a system with Poisson process characteristics for the number of UE requests participating in the connections establishment procedure in a slot. Specifically, a moderate to high access barring probability $P^B$ in conjunction with a moderately long maximum timeout $T_o^{\max}$: 1) shape the dynamics of the (overall) UE requests (from backoff and new arrivals) entering the connection establishment procedures such that Poisson model dynamics are closely approximated; and 2) increase the throughput of successful UE requests at the expense of increased mean delay for the connection establishment.

The formal result that the LTE/LTE-A uniform random backoff system can be controlled to give rise to Poisson process model dynamics can serve as basis for a wide range of future research. For instance, traffic parameter estimation can be build on this result.

## APPENDIX A
## STATIONARY DISTRIBUTION OF BACKLOG $o_n^{(r)}$ IN CIRCULATORY MODEL

In the circulatory model, arbitrary initial values $o_1^{(r)}$, $r = 1, \ldots, T$, induce a total number $N$ (24) of UE that are circulating through the backlog registers. With each increment of the time slot index $n$, the register number $r = 1$ is emptied, the registers are relabeled such that register number $r$, $r = 2, \ldots, T_o^{\max} + 1$, (see top row of registers in Fig. 1) becomes register number $r - 1$ (see bottom row of registers in Fig. 1), and register number 1 becomes register number $T_o^{\max} + 1$. The UEs from register number 1 (see rightmost register in top row in Fig. 1) are now uniformly randomly distributed over all registers, independently for each UE.

Clearly, this circulatory RA system has a unique stationary distribution as it corresponds to an irreducible, aperiodic finite-state-space Markov chain. Thus, over long time horizons $n \to \infty$, the distribution of the number $o_n^{(r)}$ of UEs in register $r$, $r = 1, \ldots, T_o^{\max} + 1$ converges to some stationary distribution. Importantly, the movement of the UEs is independent. Thus, the content of register $r$ is

$$\sum_{i=1}^{N} 1_{\{\text{UE } i \text{ is in register } r\}}, \qquad r = 1, \ldots, T_o^{\max} + 1. \tag{29}$$

Note that, for a given fixed register $r$, the terms in this sum are independent. The probability of UE $i$ to be in register $r$ is thus given by a stationary distribution.

We analyze the stationary distribution of a UE to be in register $r$. We denote $\alpha_{n,s}(r)$ for the probability of a given UE that started in register $s$ to be in register number $r$ at time $n$. The system equations (22) and (23) imply the recurrence

$$\alpha_{n,s}(r) = \alpha_{n-1,s}(r+1) + \frac{\alpha_{n-1,s}(1)}{T_o^{\max} + 1}, \quad r = 1, \ldots, T_o^{\max} \tag{30}$$

$$\alpha_{n,s}(T_o^{\max} + 1) = \frac{\alpha_{n-1,s}(1)}{T_o^{\max} + 1}. \tag{31}$$

These recurrence relationships follow by noting that a UE that is in register $r$ at time $n$ has to come either from register $r + 1$ (where it was with probability $\alpha_{n-1,s}(r+1)$ and has no other chance than moving to register $r$) or it comes from register 1 (where it was probability $\alpha_{n-1,s}(1)$) and was randomly (with probability $1/(T_o^{\max} + 1)$) assigned to register $r$. The argument for $r = T$ is analogous. We know *a priori* that $\alpha_{n,s}(r)$ converges for $n \to \infty$ to some number $\alpha_\infty(r)$, which is the invariant distribution of the Markov chain. Taking limits in (30) and (31) shows that

$$\alpha_\infty(r) = \alpha_\infty(r+1) + \frac{\alpha_\infty(1)}{T_o^{\max} + 1}, r = 1, \ldots, T_o^{\max} \tag{32}$$

$$\alpha_\infty(T_o^{\max} + 1) = \frac{\alpha_\infty(1)}{T_o^{\max} + 1}. \tag{33}$$

Further, for any $s$ and $n$, $\sum_{r=1}^{T_o^{\max}+1} \alpha_{n,s}(r) = 1$ because the UE has to be in some register. Taking the $n \to \infty$ limit gives

$$\sum_{r=1}^{T_o^{\max}+1} \alpha_\infty(r) = 1. \tag{34}$$

The only solution to (32)– (34) is

$$\alpha_\infty(r) = \frac{2(T_o^{\max} - r + 2)}{(T_o^{\max} + 1)(T_o^{\max} + 2)}, r = 1, \ldots, T_o^{\max} + 1. \tag{35}$$

## APPENDIX B
## CORRELATION OF BACKLOG $o_n^{(r)}$ AND $o_{n+1}^{(r)}$

In this appendix, we evaluate the correlation between the number $o_n^{(1)}$ of backlogged UE rejoining the connection establishment procedures in slot $n + 1$ with the number $o_{n+1}^{(1)}$ rejoining in the next slot $n + 2$ in the circulatory model. We evaluate the correlation $\mathrm{corr}(o_n^{(1)}, o_{n+1}^{(1)}) = \mathrm{cov}(o_n^{(1)}, o_{n+1}^{(1)}) / \sqrt{\mathbf{V} o_n^{(1)} \mathbf{V} o_{n+1}^{(1)}}$. We note from (29) that

$$o_n^{(1)} = \sum_{i=1}^{N} 1_{\{\text{UE } i \text{ is in register 1 in slot } n\}} =: \sum_{i=1}^{N} 1_{\{\text{UE } i,1,n\}}. \tag{36}$$

We first evaluate the covariance

$$\mathrm{cov}(o_n^{(1)}, o_{n+1}^{(1)}) = N \left( \frac{\alpha_\infty(1)}{T_o^{\max} + 1} - [\alpha_\infty(1)]^2 \right). \tag{37}$$

We analogously find the same covariance for random variables $o_n^{(1)}$ and $o_{n+\eta}^{(1)}$ that are spaced $\eta$, $\eta \geq 1$, slots apart. Moreover

$$\mathbf{V} o_{n+1}^{(1)} = \mathbf{V} o_n^{(1)} = N \alpha_\infty(1)[1 - \alpha_\infty(1)]. \tag{38}$$

Thus, we obtain the correlation

$$\mathrm{corr}(o_n^{(1)}, o_{n+1}^{(1)}) = \frac{-1}{T_o^{\max} + 1} \tag{39}$$

which converges to zero for large maximum timeout ($T_o^{\max} \to \infty$). From this vanishing negative correlation of the sequence of stationary random variables $o_n^{(1)}$, $o_{n+1}^{(1)}, \ldots$ follows asymptotic independence [39].

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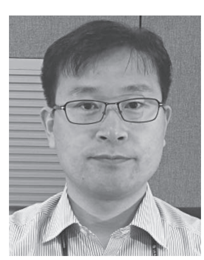

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