Journal of Educational Psychology
2011, Vol. 103, No. 1, 32–47

0022-0663/11/$12.00 DOI: 10.1037/a0021995

Teaching With Concrete and Abstract Visual Representations: Effects on Students' Problem Solving, Problem Representations, and Learning Perceptions

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In 3 experiments, we examined the effects of using concrete and/or abstract visual problem representations during instruction on students' problem-solving practice, near transfer, problem representations, and learning perceptions. In Experiments 1 and 2, novice students learned about electrical circuit analysis with an instructional program that included worked-out and practice problems represented with abstract (Group A), concrete (Group C), or abstract and concrete diagrams (Group AC), whereby the cover stories were abstract in Group A and concrete in Groups C and AC. Experiment 3 added a 4th condition (C-A) with a concrete cover story and abstract diagrams. Group AC outperformed Groups A and C on problem-solving practice in Experiments 1 and 2 and outperformed Group C on transfer across the 3 experiments; Group AC also outperformed Group C-A in Experiment 3. Further, Group A outperformed Group C on transfer in Experiments 2 and 3 and outperformed Group C-A in Experiment 3. Transfer scores were positively associated with the quality of the diagrams and the number of abstract representations drawn during the transfer test. Data on students' learning perceptions suggest that the advantage of Group AC relies on the combined cognitive support of both representations. Our studies indicate that problem solving is fostered when learners experience concrete visual representations that connect to their prior knowledge and are enabled to use abstract visual representations.

Keywords: abstract visual representation, concrete visual representation, problem solving

How can teachers help novice students develop problem-solving skills and positive learning perceptions? Research in well-structured domains, such as physics and mathematics, has shown that one promising method consists of initially demonstrating the problem-solving process with worked-out examples and later engaging learners in guided problem-solving practice (Atkinson, Derry, Renkl, & Wortham, 2000; Koedinger & Aleven, 2007; Renkl, 2005; Renkl & Atkinson, 2003). In past studies with novice college students, we found that this method can successfully promote the transfer of learned principles to novel electrical engineering problems (Moreno, Reisslein, & Delgoda, 2006; Moreno, Reisslein, & Ozogul, 2009a). Despite these favorable outcomes, the assumptions underlying the design of the visual representations corresponding to the worked-out and practice problems (e.g., electrical circuit diagrams) were not tested.

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This article is based on work supported by the National Science Foundation under Grant 0648568. Any opinions, findings, and conclusions or recommendations expressed in this article are those of the authors and do not necessarily reflect the views of the National Science Foundation. This work is presented in memory of Roxana Moreno who passed away on July 24, 2010. She was a great colleague and inspired many.

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The design of the visual representations in our past studies was based on the typical diagrams used in college engineering textbooks, which mostly represent electrical engineering problems with abstract diagrams. An abstract diagram provides a schematic depiction of an engineering problem and uses standard engineering symbols (Alexander & Sadiku, 2004; Irwin & Nelms, 2005), such as the zig-zag symbol to represent a light bulb. In contrast, we define concrete electrical engineering diagrams as those that provide realistic illustrations of the real-life electrical elements described in the word problems (e.g., the image of a light bulb), instead of the standard symbols used by engineers. The purpose of the present research was to investigate whether representing engineering problems with abstract, concrete, or a combination of abstract and concrete diagrams would affect students' problem-solving practice, near transfer, problem representations, and learning perceptions.

Several studies have compared how students learn from schematic versus realistic representations of science diagrams (Butcher, 2006; Dwyer, 1968, 1969; Joseph & Dwyer, 1984) and science simulations (Goldstone & Sakamoto, 2003; Goldstone & Son, 2005; Scheiter, Gerjets, Huk, Imhof, & Kammerer, 2009). In addition, past research has compared learning from different problem representations, such as diagrams versus verbal descriptions of math and physics problems (Larkin & Simon, 1987), electricity diagrams versus equations (Cheng, 2002), graphics versus sentential representations of logical reasoning (Stenning, Cox, & Oberlander, 1995), or grounded (e.g., story problems) versus symbolic (e.g., equations) representations of algebra (Koedinger, Alibali, & Nathan, 2008; Koedinger & Nathan, 2004). Yet, to our knowledge, there are no empirical studies that have examined the role of the concreteness of visual representations in problem solving.

In a recent preliminary study conducted with gifted high-school students (Moreno, Reisslein, & Ozogul, 2009b), students who learned about electrical circuit analysis with abstract diagrams produced higher transfer scores and better problem representations after instruction than those who learned with diagrams that included lifelike images of the circuit elements. The purpose of the curent research is to extend this preliminary study by including high-school students in regular classrooms (Experiments 1 and 2) and college students (Experiment 3) as participants and by adding an instructional treatment in which abstract and concrete visual problem representations were combined.

Role of Visual Representations in Problem Solving

One approach is to develop a general problem-solving model that can be applied to a diversity of problem categories and problem domains (Anderson, 1993; Bransford & Stein, 1984; Gick, 1986; Hayes, 1988). A contemporary model has the following five steps: (a) identifying the problem, (b) representing the problem, (c) selecting a strategy, (d) implementing the strategy, and (e) evaluating the problem solution (Bruning, Schraw, Norby, & Ronning, 2004).

Studies have documented the central role that the problem representation step plays in redescribing problems in terms of concepts and principles learned (Chi, Feltovich, & Glaser, 1981), reducing the difficulty of the problem-solving process (Newell & Simon, 1972; Simon & Hayes, 1976), and promoting the solution of problems (Brenner et al., 1997; Collins & Ferguson, 1993; Mayer & Hegarty, 1996; Rittle-Johnson, Siegler, & Alibali, 2001; Zhang, 1997). Furthermore, expertise studies in a variety of domains have shown that visual representations are a fundamental tool to support the reasoning and problem solving of experts (Ericsson & Smith, 1991; Knorr-Cetina & Amann, 1990; Kozma, Chin, Russell, & Marx, 2000; Lynch, 1995; Lynch & Woolgar, 1990; Roth, Bowen, & McGinn, 1999). After conducting a set of naturalistic studies about the representational expertise of chemists, Kozma (2003) concluded,

> The first thing we noticed was that representations were everywhere in these laboratories. Structural diagrams and equations were written on flasks and vials filled with compounds being heated, filtered, or waiting for reactions. They were written on glass hoods and white boards throughout the lab. And they were in notebooks and reference books, and in journal articles and advertisements on bookshelves and bench tops. (p. 209)

In line with these findings, Schank (1994) identified the use of visual representations as one of the seven essential skills of professional biologists.

The way in which visual representations are designed in instruction can play an important role in the lived experience of students and in their associated appropriation of effective graphing practices (Collins & Ferguson, 1993; Day, 1988; Kirshner, 1989; Zhang, 1997). Although diagrams have been shown to support reasoning and problem solving (Glasgow, Narayanan, & Chandrasekaran, 1995), the evidence that not all visual representations are equally helpful to all students suggests that their level of concreteness may have an effect on learning (Butcher, 2006). In the next sections, we discuss three hypotheses for reasoning about the ways in which abstract and concrete visual representations might support the problem solving of novice students.

The Case for Concrete Visual Representations

Concrete visual representations are those that illustrate the real-life objects corresponding to a problem's cover story. For example, the middle part of Figure 1 illustrates the battery and light bulbs of the following concrete problem: "Consider two light bulbs with resistance values $R_1 = 6\ \Omega$ and $R_2 = 12\ \Omega$ connected in parallel to a $V = 9$ V battery. Find the total resistance of this circuit." Educators and cognitive scientists have frequently noted the value of concrete representations in education. In one study, 84% of secondary math teachers reported believing that concrete representations can facilitate learning significantly (Perry, Howard, & Tracey, 1999). The main thesis underlying this position is that even if the instructional goal is to develop abstract knowledge in a domain, abstractions can be most effectively learned through experience with perceptually rich, concrete knowledge representations (Goldstone & Sakamoto, 2003). More specifically, realistic rendering of objects within a representation may benefit students by making a problem more readily accessible in long-term memory (DiFonzo, Hantula, & Bordia, 1998; Koedinger & Nathan, 2004). Realistic diagrams depict a close correspondence between diagrams and the concrete objects that they represent; therefore, realistic diagrams rely less on knowledge conventions for their interpretations. According to this view, the usefulness of abstract diagrams is limited because students must be able to understand and make use of abstract visual conventions to correctly interpret the problem representation (Hegarty, Carpenter, & Just, 1991).

The research on how individuals learn suggests that when students are novices in a domain, their early encounters with new concepts and principles to be learned should be designed to activate and build on their existing knowledge (Bransford, Brown, & Cocking, 2000; Collins, Brown, & Newman, 1989; Donovan & Bransford, 2005). In a concrete electrical engineering problem representation, the electrical components of a circuit (i.e., lightbulbs, batteries) illustrated in diagrams are literally from the students' everyday lives. These iconic representations can be used as the foundation to construct a meaningful model of the problem scenario. Once this meaning is established, then students should find it easier to advance toward thinking about how the electrical principles learned can be used to solve the specific problem (Sharp & Adams, 2002). Moreover, it has been argued that realistic graphics may promote higher motivation because they are more interesting and evocative, a reason put forward in support of the design of virtual reality learning environments (Goldstone & Son, 2005; Grady, 1998; Heim, 2000).

The Case for Abstract Visual Representations

Abstract visual representations are those that use conventional symbols to represent the relevant elements of a problem's cover story. For example, the top portion of Figure 1 illustrates the conventional circuit symbols of the following problem: "Consider two resistors with resistance values $R_1 = 6\ \Omega$ and $R_2 = 12\ \Omega$ connected in parallel to a voltage source with $V = 9$ V. Find the total resistance of this circuit." Although concrete visual represen-

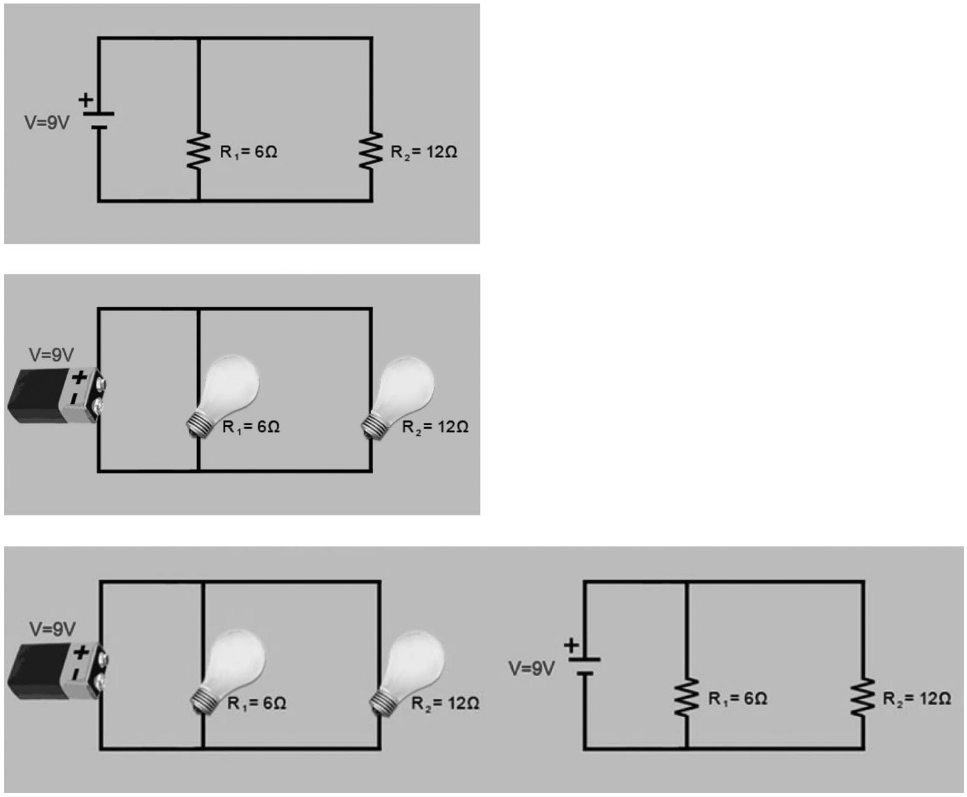


Figure 1. A circuit ciagram sample from the instructional program used by Groups A (top), C (middle), and AC (bottom).

tations may have the cognitive advantage of relying less on knowledge conventions and the motivational advantage of being more interesting than more abstract ones, they have limited referential flexibility. Specifically, the knowledge contained in more realistic representations is found to have limited portability and transfer (Bassok & Holyoak, 1989; Sloutsky, Kaminski, & Heckler, 2005). In addition, a strong argument supporting the use of abstract visual representations is that concrete visual representations divert novice students' attention to irrelevant problem information. For instance, Dwyer (1968, 1969; Joseph & Dwyer, 1984) were able to conclude from their investigations about learning with realistic versus diagrammatic science representations that the realistic cues of concrete representations acted as a cognitive distraction. Several other studies have found that more realistic visual representations unintentionally focus students' attention on superficial information at the expense of information that is necessary to accomplish the learning objectives (Gianutsos, 1994; Hegarty & Kozhevnikov, 1999; Sloutsky, Kaminski, & Heckler, 2005).

As noted by several researchers (DeLoache, 2000; DeLoache & Marzolf, 1992; Goldstone & Son, 2005; Markman & Gentner, 1993), individuals are more likely to respond on the basis of superficial object attributes as the richness of the objects in visual representations increases. By leaving out the details of concrete objects, abstract visual representations help students focus on the relevant structural characteristics of the problem (Colin, Chauvet, & Viennot, 2002; Elia, Gagatsis, & Demetriou, 2007). In the few studies that found an advantage for learning with concrete representations, the concrete information conveyed by the instructional materials was relevant to the learning objectives of the lesson (Goldstone & Sakamoto, 2003).

An additional argument in support of abstract visual representations is that they put fewer demands on working memory during problem solving (Koedinger, Alibali, & Nathan, 2008). Specifically, abstract visual representations do not require students to keep track of their referents while solving the problem, which allows students to more easily imagine manipulations of quantitative relations.

The Case for Combining Abstract and Concrete Visual Representations

The previous two sections pointed out advantages and disadvantages to both concrete and abstract visual problem representations. Therefore, the third hypothesis tested in this research is that instruction that includes simultaneous concrete and abstract visual representations will more effectively promote students' problem solving than learning with abstract or concrete representations alone. Combined abstract and concrete visual representations are those that both illustrate the real-life objects corresponding to a problem's cover story and use conventional symbols to represent the relevant elements of a problem's cover story. It can be argued that students who learn with both representations benefit from the perceptual scaffolding, schema activation, and motivational benefits that concrete visual representations offer while at the same time they are provided with a tool to generalize the common underlying structure of problems that are superficially dissimilar.

Although there are no studies to our knowledge that have examined the simultaneous combination of abstract and concrete visual representations for problem solving in a well-defined domain, such as engineering, a couple of recent studies have manipulated the order in which students learn with schematic (abstract) and realistic (concrete) simulations. In one study (Scheiter et al., 2009, Experiment 2), students who watched a realistic simulation followed by a second realistic simulation showed significantly lower learning outcomes than those who learned with two successive schematic simulations: a realistic simulation followed by a schematic simulation or a schematic simulation followed by a realistic one. Another study compared the amount of transfer between two simulations that were governed by the same principle, with the simulation elements remaining concrete, remaining abstract, or switching from concrete to abstract and vice versa (Goldstone & Son, 2005). The findings showed that the best transfer was observed when concrete elements became abstracted at a later stage. Table 1 lists the three visual representation hypotheses discussed, the cognitive advantages for each, and the predictions to be tested in the present study.

Purpose of the Study

Our purpose in this research was to test the predictions derived from the three hypotheses discussed in the previous sections. To do this, we used electrical circuit problem solving as the task domain and test bed. To this end, we examined the effects of learning with concrete, abstract, or both types of visual representations on students' problem-solving practice, transfer, visual representations, and learning perceptions. Visual representations were presented in the form of electrical circuit diagrams. In addition, we investigated the extent of the correlation between students' visual representations and problem-solving transfer. More specifically, the present study was motivated by the following research questions:

1. Does the type of diagram used during instruction affect problem-solving transfer?

2. Does the type of diagram used during instruction affect the quality and type of diagrams produced during problem solving?

3. Is there a correlation between the quality of students' diagrams and their problem-solving transfer scores?

4. Does the type of diagram used during instruction affect students' perceptions about the diagrams?

To answer these questions, we conducted three experimental studies. In Experiment 1, participants learned with abstract cover stories and abstract visual representations (diagrams) of the problems (Group A), concrete cover stories and visual representations of the problems (Group C), or concrete cover stories and a combination of abstract and concrete visual representations of the problems (Group AC). Experiment 2 was identical to the first study except that we examined the type of diagrams that students spontaneously produced when solving new problems after instruction. Finally, in Experiment 3, we added a fourth condition to those used in the prior two studies in which students learned with concrete cover stories and abstract visual representations of the problems. The four instructional conditions tested in our study correspond to those found in typical precollege and college instructional materials (Alexander & Sadiku, 2004; Irwin & Nelms, 2005; Orsak et al., 2004). Learning was measured with a paper-and-pencil near-transfer test in which students were asked to solve a set of novel electrical circuit problems applying the electrical principles learned, and learning perceptions were measured with an open-ended question in which students were asked to describe what they liked about the instructional program with which they learned.

Experiment 1

Method

Participants and design. The participants were 71 high-school students (37 female and 34 male) who lacked formal

Table 1
Three Visual Representation Hypotheses

Hypothesis	Cognitive advantages	Predictions
Concrete visual representations promote better problem solving than abstract visual representations.	Making a problem more readily accessible in long-term memory by activating and building on students' existing knowledge. Promoting motivation to learn by presenting interesting realistic graphics rather than conventional symbols.	Students who learn with concrete cover stories and corresponding concrete visual representations will outperform their counterparts on problem-solving transfer.
Abstract visual representations promote better problem solving than concrete visual representations.	Focusing students on the relevant, structural characteristics of the problem rather than on the superficial details of concrete visual representations. Providing a tool to generalize the common underlying structure of problems that are superficially dissimilar.	Students who learn with abstract cover stories and corresponding abstract visual representations will outperform their counterparts on problem-solving transfer.
Combined abstract and concrete visual representations promote better problem solving than abstract or concrete visual respresentations alone.	Providing the combined cognitive advantages that concrete and abstract visual representations offer.	Students who learn with concrete cover stories and corresponding abstract and concrete visual representations will outperform their counterparts on problem-solving transfer.

education in electrical circuit analysis. The mean age of the participants was 13.73 years ($SD = 0.63$ years). Of the students, 42 reported that they were Caucasian, 13 reported that they were Hispanic American, 2 each reported that they were African American, Asian American, or Native American, and 10 reported that they were from other or multiple ethnicities. There were 24 students each in Groups A and AC and 23 students in the Group C.

Learning materials. The learning materials consisted of an interactive instructional program that included the following sections: (a) a demographic survey, (b) a pretest, (c) a conceptual overview of electrical circuit analysis with a corresponding worked-out problem, (d) a problem-solving practice session, and (e) a feedback screen in which students were asked to describe what they liked the most about the instructional program.

The first section of the program consisted of a questionnaire in which students were asked to report their gender, age, and ethnicity. The second section of the program was a pretest aimed at assessing students' prior knowledge in the domain. The pretest consisted of the following six multiple-choice questions: "What is an electrical current?" "What is resistance?" "What happens if you connect two strings of Christmas lights in parallel?" "A Gameboy runs from a 9 V battery and draws a current of 0.1 A. What is the resistance of the Gameboy?" "The electric motor of a toy car has a resistance of $R = 20\ \Omega$ and requires a current of 0.4 A. What battery voltage is needed to operate the toy car?" "You connect a lamp with a resistance of $R = 3\ \Omega$ to a 6 V battery. How large is the current flow through the lamp?"

The third section of the program included a lesson about the meanings and units of electrical current, voltage, and resistance followed by a worked example showing how to use Ohm's Law to calculate the total resistance of a parallel circuit with a given source voltage and individual resistance values. In particular, the worked example demonstrated the following three problem-solving steps: (a) calculating the value of the current flowing through each individual resistor by using Ohm's Law; (b) calculating the total current flowing in the circuit by summing up the currents flowing through the individual resistors; and (c) calculating the total resistance of the parallel circuit by applying Ohm's Law.

The fourth section of the program consisted of a problem-solving practice session. It presented two electrical circuit problems in which students were asked to compute the total resistance of a parallel circuit by applying the principles they had learned in the conceptual portion of the program. The session was designed so that participants received immediate feedback after completing each solution step for the two practice problems. Specifically, if the solution to the first step was correct, the program confirmed the correctness of the solution. If the solution was incorrect, the program presented an explanation about how to solve the step correctly as well as the correct solution. After studying the explanatory feedback, students could click on the *continue* button to proceed to the next solution step while the correct solution for the preceding step remained on the screen. After completing the three steps of the first practice problem, students could click on the *next problem* button to work on the second problem.

In the A version of the program, the cover stories of the worked-out and practice problems were presented in abstract form, such as "Consider two resistors connected in parallel to a voltage source." Problems were represented with an electrical circuit diagram in which the electrical circuit elements (e.g., voltage source, resistors) were represented with the abstract symbols that are standard in the electrical engineering field. In the C version of the program, the cover stories of the worked-out and practice problems were presented in concrete form, such as "Consider two light bulbs connected in parallel to a battery." Problem representations included diagrams that were identical to those presented to the A group, except that the abstract engineering symbols were replaced by the concrete electrical elements described in the word problems. The AC version of the program was identical to the C version, except that both concrete and abstract diagrams were presented next to each other. Figure 1 shows a sample diagram corresponding to the instructional program used in the A, C, and AC treatments.

The last section of the program was aimed at investigating students' perceptions about the instructional program. More specifically, we were interested in examining whether there were significant differences among groups in the participants' perceptions of the diagrams presented during instruction. To this end, once the practice session was over, a new screen was presented with the following instruction at the top: "Please describe what you liked the most about the instructional program that you just learned from." A text box was presented below the instruction to allow students to type in their answers.

Test materials. The test materials consisted of a paper-and-pencil near-transfer test. It was designed to assess students' ability to transfer the electrical circuit principles learned to solve a set of three novel isomorphic problems that had the same underlying structure as but different surface characteristics from the worked example in the conceptual overview and the problems presented during the practice session. The test was also designed to examine the quality of the diagrams produced by the participants as they solved the problems. The three problems were as follows:

> You wire a subwoofer speaker with resistance $R_s = 16\ \Omega$ and a regular speaker with a resistance of $R_r = 8\ \Omega$ in parallel and operate this electrical circuit with a $V = 6$ V battery. What is the total resistance of this electrical circuit?
>
> The electrical system of a remote controlled toy helicopter consists of a motor with resistance $R_m = 4.5\ \Omega$ and a control unit with resistance $R_c = 72\ \Omega$. All these components are wired in parallel and are connected to a $V = 9$ V battery. What is the total resistance of this parallel electrical circuit?
>
> Your laptop has a central processing unit (CPU) with resistance $R_{cpu} = 25\ \Omega$, an LCD screen with resistance $R_{lcd} = 50\ \Omega$, and a hard drive with resistance $R_{hd} = 75\ \Omega$. All these parts are wired in parallel. The laptop's battery provides a voltage of $V = 12.5$ V. What is the total resistance of the laptop's electrical system?

The transfer test was identical in all conditions, and no time limit was imposed on the participants. After each problem statement, the following instruction was given to prompt students to draw a visual representation of the problem: "Draw a diagram for the electrical circuit described in the problem."

Scoring. The answers to the multiple-choice pretest and practice problems were recorded and scored by the computer program. One point was given for each correct answer given in the pretest, and one point was given for each correct solution step produced during the problem-solving practice session. On the basis of these data, a total pretest and practice score was computed by adding the number of correct solutions produced in each task. The maximum score students could attain in both the pretest and the practice session was six.

To score the transfer test, we developed a scoring rubric in which students were given one point for correctly calculating the electrical currents in the circuit, the total current in the circuit, and the total resistance in the circuit. Students could receive up to three points for each one of the three transfer problems, leading to a potential maximum transfer score of nine points. Using this rubric, two engineering instructors who were unaware of the experimental conditions of the participants scored the transfer test (interrater reliability = 98.5%).

To answer our second research question, we developed a coding rubric for scoring the quality of the circuit diagrams produced by the participants during the transfer test. The rubric included a list of the elements that encompass a complete representation of the electrical circuit elements and variables necessary to solve the word problem. In particular, one point was given for drawing each of the following: a closed circuit, the voltage source, the resistive elements in parallel, and the appropriate labels and numerical values for the voltage and resistors. On the basis of this rubric, the maximum diagram score that participants could achieve for the first two transfer problems (with two resistors) was 10, and the maximum diagram score that participants could achieve for the third problem (with three resistors) was 13, leading to a potential maximum score of 33 points. Two instructors who were unaware of the experimental conditions of the participants scored the diagrams produced during the transfer test (interrater reliability = 98.9%).

In addition, we developed a coding rubric for classifying the type of representations (A vs. C) that the participants drew for the voltage and resistors depicted in the transfer test problems. Concrete representations were those that included real-life drawings of the electrical elements described in the word problem. Abstract representations were those that did not show any attempt to draw the real-life elements described in the word problem. Once the representations were classified into abstract or concrete, total abstract and concrete representation scores were calculated for each participant by adding the number of abstract and concrete drawings, respectively, produced during the transfer test. Participants could receive up to three points for each representation type (A or C) produced in the first two transfer problems (with one voltage source and two resistors) and could receive up to four points for each representation type produced in the third problem (with one voltage source and three resistors), leading to a potential maximum score of 10 points for the total number of A and C representations. Two instructors who were unaware of the experimental conditions of the participants scored the type of representations produced during the transfer test (interrater reliability = 97%).

Finally, we developed response categories based on the content of participants' written responses to the open-ended statement presented at the end of the instructional program: "Please describe what you liked the most about the instructional program that you just learned from." The categories included the program's easiness (i.e., "The program was easy to understand," "It made it easy to learn"), examples (i.e., "I liked how there were many examples and after a few of them I understood how to solve the problem," "I liked the example problems because they helped me to understand it better"), feedback (i.e., "How it showed you how to do the problem if you messed up," "I liked that we could check our answers and see if we got it right or wrong"), formulas (i.e., "I liked the different formulas given," " I liked that the program gave you the formulas"), entertainment and novelty value (i.e., " It was much funner than normal classes," "It was something different from everyday school work"), auditory guidance ("I liked that there was someone talking to us while trying to learn it. I thought it was really helpful," "That they talked you through it"), diagrams (i.e., "What I liked best about the program was the pictures," "The pictures really added to the learning experience"), and delivery medium (i.e., "I liked that we got to do it in computers," "I liked that it was on a personal laptop"). Students also gave school avoidance responses (i.e., "I liked that I didn't have to do the classroom work," "I liked getting to do this instead of biology"), underspecified responses (i.e., "I liked everything about it," "I liked all of it"), and blank responses. Two instructors who were unaware of the experimental conditions of the participants coded students' responses to the open-ended question (interrater reliability = 95.8%). Because of the focus of this research, we tallied for each participant whether or not a specific positive comment about the program's diagrams was made.

Software and apparatus. The computer program used in the study was developed with Adobe Flash CS3 software, an authoring tool for creating Web-based and stand-alone multimedia programs. The apparatus consisted of a laptop computer system, with a screen size of $1{,}680 \times 1{,}050$ pixels, and headphones.

Procedure. After providing the experimenter with their consent, the participants were randomly assigned to one of the three treatment conditions (A, C, or AC) and seated in front of a laptop. Students were told that they would be working individually and were instructed to put on their headphones and initiate the instructional program. Once the instructional program was over, students were provided with the paper-and-pencil transfer test. After completing the test, the participants were debriefed and thanked for their participation.

Results

Statistical assumptions were evaluated and met. In all statistical tests, alpha was set at .05, and Tukey post hoc tests were conducted as follow-up analyses.

Preliminary analyses. We first examined whether there were significant differences between the groups on the pretest scores with a univariate analysis of variance (ANOVA) that included treatment condition as a variable and students' pretest score as a dependent variable. The analysis did not reveal significant differences among the groups, $F(2, 68) = 0.05$. Because the practice section of the program was self-paced, we also examined whether there were significant differences among the groups in the time spent on this task. An ANOVA with treatment condition as a variable and the time on task as a dependent variable did not reveal significant differences among the groups on this measure, $F(2, 68) = 0.48$. Table 2 shows the means and standard deviations for the three groups on all outcome measures.

Research Question 1: Does the type of diagram used during instruction affect problem-solving transfer? To answer this question, we first examined students' performance during the practice session. To this end, we conducted an ANOVA with

Table 2
Means and Standard Deviations of Outcome Measures for Three Groups: Experiment 1

	Group					
	A		C		AC	
Measure	*M*	*SD*	*M*	*SD*	*M*	*SD*
Practice time (min)	18.82_{a}	3.36	18.11_{a}	2.16	18.29_{a}	1.97
Pretest	2.00_{a}	0.83	2.09_{a}	0.95	2.04_{a}	1.04
Practice	2.04_{a}	1.83	1.83_{a}	1.40	3.04_{b}	1.63
Transfer	$3.07_{a,b}$	3.40	1.66_{b}	2.39	4.23_{a}	3.64
Diagram quality	17.15_{a}	8.77	15.95_{a}	8.65	17.35_{a}	10.32
A representations	4.82_{a}	3.13	0.45_{b}	0.89	2.78_{a}	3.46
C representations	0.17_{a}	0.64	3.52_{b}	3.49	1.42_{a}	2.84
Diagram perceptions	0.04	0.20	0.22	0.42	0.21	0.41

Note. A, C, and AC stand for abstract, concrete, and abstract and concrete, respectively. Means in the same row that do not share subscripts are significantly different.

treatment condition as between-subjects variable and the practice session score as a dependent variable. The analysis revealed a significant, medium-size treatment effect, $F(2, 68) = 3.75$, $MSE = 9.98$, $p < .03$, $\eta^2 = .10$. Post hoc tests showed that the AC group outperformed the A and C groups on the practice session (*p*s = .04 and .01, respectively). There were no other significant differences among the groups.

Second, we conducted an ANOVA with treatment condition as a between-subjects variable and the transfer test score as a dependent variable. The analyses showed a significant, medium-size treatment effect, $F(2, 68) = 3.81$, $MSE = 39.06$, $p = .03$, $\eta^2 = .10$. Post hoc tests revealed that the AC group outperformed the C group on the transfer measure ($p = .02$). No other significant differences were noted.

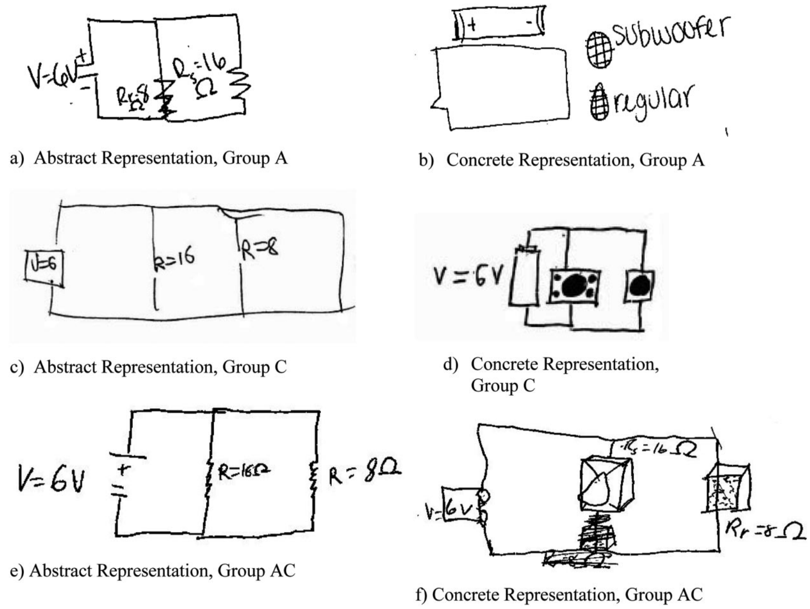
Research Question 2: Does the type of diagram used during instruction affect the quality and type of diagrams produced during problem solving? Sixty students (84.5%) attempted to draw at least one diagram during the posttest. A one-sample chi-square test showed that the percentage of participants that drew diagrams did not differ by condition, $\chi^2(2, N = 71) = 5.88$, $p > .05$. To examine whether the type of diagram used during instruction affected the quality of diagrams that students produced during problem solving, we conducted an ANOVA using treatment condition as between-subjects variable and students' diagram scores as a dependent measure. No significant differences among treatments were found, $F(2, 68) = 1.12$.

To examine whether the type of diagram used during instruction affected the type of diagrams that students produced during problem solving, we first conducted an ANOVA with treatment condition as a between-subjects variable and the number of abstract representations produced in the transfer test as a dependent measure. The results revealed a significant, large treatment effect, $F(2, 68) = 7.06$, $MSE = 57.99$, $p = .002$, $\eta^2 = .17$. The A and AC groups produced significantly more abstract representations than the C group ($p = .001$ and $p = .008$, respectively). Second, we conducted an ANOVA with treatment condition as a between-subjects variable and the number of concrete representations produced in the transfer test as a dependent measure. The results showed a significant, large treatment effect, $F(2, 68) = 9.89$, $MSE = 67.32$, $p = .000$, $\eta^2 = .22$. Group C produced significantly more concrete representations than Groups A and AC ($p = .000$ and $p = .007$, respectively).

Research Question 3: Is there a correlation between the quality and type of diagrams produced and students' problem-solving transfer scores? A positive correlation was found between students' diagram scores and their transfer scores (Pearson $r = .33$, $p = .005$). In addition, we found a positive correlation between the number of abstract representations drawn by the participants and their transfer test scores (Pearson $r = .24$, $p < .05$) but no significant correlation between the number of concrete representations drawn by the participants and their transfer test scores (Pearson $r = -.03$, $p = ns$).

Samples of participants' diagram drawings are shown in Figure 2. The sample abstract representation by a learner in Group A (Figure 2A) represents the given circuit as an electrical engineering expert would represent it. On the other hand, the sample concrete representation by a Group A learner (Figure 2B) and the sample concrete representation by a Group C learner (Figure 2D) are of little use for problem solving. The sample abstract representation by a Group C learner (Figure 2C) contains the relevant given circuit parameters (although the overall circuit structure is incorrectly drawn) and reveals how the learner, who had not experienced the abstract symbols for circuit elements in the instructional program, had to develop his or her own symbols for abstract representation. The sample representations by Group AC learners (Figure 2E and 2F) correctly represent the given circuit; an expert electrical engineer would prefer the abstract representation as it avoids the irrelevant details of the circuit elements.

Research Question 4: Does the type of diagram used during instruction affect students' perceptions about the diagrams? Using the data from the qualitative analysis of students' open responses to the question asked at the end of the instructional program, comparisons among groups were made on the number of positive comments that the participants made about the program's diagrams. An ANOVA with treatment condition as a between-subjects variable and the number of positive diagram comments as a dependent measure did not reveal a significant difference among groups, $F(2, 68) = 1.39$.



a) Abstract Representation, Group A
b) Concrete Representation, Group A
c) Abstract Representation, Group C
d) Concrete Representation, Group C
e) Abstract Representation, Group AC
f) Concrete Representation, Group AC

Figure 2. Sample abstract and concrete representations produced by participants in three groups.

Discussion

The findings of Experiment 1 support the hypothesis that instruction that presents worked-out and practice problems with a combination of abstract and concrete representations promotes better learning than instruction with abstract or concrete representations alone. This was demonstrated by the practice session results, which showed that Group AC outperformed Groups A and C, and by the transfer test results, which showed that Group AC outperformed Group C. The fact that students' positive comments about the program diagrams did not differ among groups suggests that the advantage of Group AC relied on cognitive rather than motivational causes.

In addition, students' diagram scores and number of abstract representations were positively correlated with their transfer scores, and Groups AC and A drew significantly more abstract representations during the transfer test. These findings suggest that the use of abstract problem representations during instruction (either alone or in combination with concrete representations) plays an important role in facilitating students' independent problem solving.

In summary, the findings of Experiment 1 support the hypothesis that novice students who are in the beginning stage of problem-solving skill acquisition should be presented with both abstract and concrete graphic representations during instruction. According to this hypothesis, the cognitive support of concrete representations is to promote novice students' understanding by relating the superficial features of the problems to their prior knowledge, whereas the support of abstract representations is to show students how to represent the referents of the cover story in a generalizable way, thereby helping students focus their attention on relevant structural problem information.

Experiment 2

The goal of Experiment 2 was to replicate the first study with a different group of students and to examine whether learning in A, C, and AC conditions would affect the likelihood of spontaneously drawing diagrams during problem solving. Previous experimental and naturalistic studies comparing the use of representations of experts and novices in the domain of chemistry have shown that novices do not spontaneously generate representations to help them think about science problems (Kozma, 2003; Kozma & Russell, 1997). Experiment 2 extends this research to the domain of electrical engineering problem solving.

Method

Participants and design. The participants were 128 high-school students (79 female and 49 male) who lacked formal education in electrical engineering. The mean age of the participants was 15.43 years (SD = 1.41 years). Of the students, 62 reported being Hispanic American, 39 reported being Caucasian, 8 reported being African American, 4 reported being Asian American, 3 reported being Native American, and 12 reported multiple or other ethnicities. There were 41 participants in Group A, 44 participants in Group C, and 43 participants in Group AC.

Materials and apparatus. The materials and apparatus were the same as those described in Experiment 1, except that we deleted the explicit instruction to draw a diagram from the paper-and-pencil transfer test. This was done to examine whether and what type of diagrams students would draw during the transfer test without being prompted to do so.

Procedure. The procedure was identical to the one used in Experiment 1.

Results

The statistical analyses and tests were the same as those used in Experiment 1. Similar to the results of Experiment 1, no significant differences among the treatment groups were found on the pretest or on the time spent on the practice session of the program, $F(2, 125) = 0.59$ and $F(2, 125) = 2.40$, respectively. Table 3 shows the means and standard deviations corresponding to all outcome measures for Groups A, C, and AC.

Research Question 1: Does the type of diagram used during instruction affect problem-solving transfer? To answer this question, we first conducted an ANOVA with treatment condition as a between-subjects variable and the practice session score as a dependent variable. The analysis revealed a significant, small treatment effect, $F(2, 125) = 3.59$, $MSE = 8.63$, $p = .03$, $\eta^2 = .05$. Post hoc tests showed that Group AC outperformed Group C ($p = .013$) and Group A ($p = .04$) on the practice session. Second, we conducted an ANOVA with treatment condition as a between-subjects variable and the transfer test score as a dependent variable. The analyses showed a significant, medium-size treatment effect, $F(2, 125) = 6.55$, $MSE = 88.20$, $p = .002$, $\eta^2 = .09$. Post hoc tests revealed that Group AC outperformed Group C ($p = .001$) and that Group A outperformed Group C ($p = .02$) on the transfer measure. No other significant differences were noted.

Research Question 2: Does the type of diagram used during instruction affect the quality and type of diagrams produced during problem solving? Only 19 students (14.84 %) spontaneously drew diagrams while solving the transfer test problems. A one-sample chi-square test revealed that the percentage of participants that drew diagrams did not differ by condition, $\chi^2(2, N = 128) = 0.90$, $p > .05$. Because of the low number of spontaneous diagrams produced during the transfer test, statistical comparisons on students' diagram scores and abstract and concrete representations were not conducted. We note that students in Groups AC and A produced only abstract diagram representations and that students in Group C produced only concrete diagram representations.

Research Question 3: Is there a correlation between the quality and type of diagrams produced and students' problem-solving transfer scores? Similar to Experiment 1, there was a positive, significant correlation between students' diagram scores and their transfer scores (Pearson $r = .31$, $p = .002$). In addition, a nearly significant, positive correlation between the mean number of abstract representations drawn by the participants and their respective transfer test scores was noted (Pearson $r = .42$, $p = .07$). No significant correlation was found between the mean number of concrete representations drawn and participants' transfer test scores (Pearson $r = -.03$, $p = ns$).

Research Question 4: Does the type of diagram used during instruction affect students' perceptions about the diagrams? Similar to Experiment 1, an ANOVA with treatment condition as a between-subjects variable and the number of positive diagram comments as a dependent measure did not reveal a significant difference among groups, $F(2, 125) = 1.64$.

Discussion

The findings from Experiment 2 replicate the pattern of results found in Experiment 1 in the following ways. First, we found that Group AC outperformed Groups C and A during the practice session. Second, Group AC outperformed Group C on the transfer measure. A difference between the findings of the two studies is that Group A in Experiment 2 also outperformed Group C on the transfer measure. This result is consistent with the results of a prior study in which comparisons on a transfer test were made between gifted high-school students who learned with abstract versus concrete diagrams (Moreno et al., 2009b). Third, in line with the results from Experiment 1, the participants in Groups A and AC who spontaneously drew diagrams during the transfer test chose to use the conventional engineering symbols to represent the transfer problems, whereas Group C chose to represent the transfer problems with real-life drawings. Fourth, Experiment 2 showed a positive correlation between the quality of the diagrams drawn and students' transfer scores. Finally, similar to Experiment 1, there were no significant differences among groups on students' positive perceptions about the program's diagrams.

In summary, the findings of Experiment 2 further support the hypothesis that novice students who are in the beginning stage of problem-solving skill acquisition should be presented with both abstract and concrete visual representations during instruction. The finding that the groups did not differ on the number of positive diagram perceptions reported by the end of the learning experience suggests that the transfer advantage of Group AC originates from

Table 3
Means and Standard Deviations of Outcome Measures for Three Groups: Experiment 2

	Group					
	A		C		AC	
Measure	*M*	*SD*	*M*	*SD*	*M*	*SD*
Practice time (min)	19.83_a	4.33	19.99_a	4.96	18.13_a	3.63
Pretest	2.12_a	0.87	2.27_a	1.00	2.35_a	1.00
Practice	2.95_a	1.56	2.82_a	1.63	3.65_b	1.44
Transfer	4.86_a	3.78	2.96_b	3.78	5.74_a	3.44
Diagram quality	24.17	10.11	14.00	10.86	16.25	9.62
A representations	8.33	3.26	0.00	0.00	4.37	3.38
C representations	0.00	0.00	4.20	3.11	0.00	0.00
Diagram perceptions	0.07	0.26	0.14	0.35	0.05	0.21

Note. A = abstract; C = concrete; AC = abstract and concrete. Means in the same row that do not share subscripts are significantly different.

the combined cognitive support that each representation offers rather than from the motivational value of the realistic diagrams.

An important point to note in both experiments is that the transfer test included concrete, real-life electrical circuit elements in the cover stories described by the word problems. Because Group A (unlike Groups AC and C) was provided with abstract cover stories during instruction but concrete cover stories in the transfer test, it is remarkable that the performance of this group was not statistically different from that of Group C in Experiment 1 and was significantly superior to Group C in Experiment 2. On the basis of the well-known phenomenon of encoding specificity (Tulving & Osler, 1968), Group C should have outperformed Group A simply because the conditions of retrieval (i.e., transfer test) were identical to those of encoding (i.e., instruction). This observation raised the following question: What are the effects of combining concrete word problems with corresponding abstract diagrams on students' problem solving, diagram representations, and perceptions? The objective of Experiment 3 was to answer this question.

Experiment 3

The goals of Experiment 3 were to examine whether the results of the first two experimental studies would replicate with a group of older students who were also novices in the domain and to examine whether learning with worked-out and practice problems that combine concrete cover stories and corresponding abstract diagrams would affect students' problem-solving practice, transfer, visual representations, or learning perceptions. To this end, we added a fourth condition to the third study. To help distinguish between the two conditions that included abstract representations in their diagrams, we labeled the four conditions as follows: A-A (abstract cover stories plus abstract diagrams, similar to the A condition in Experiments 1 and 2); C-C (concrete cover stories plus concrete diagrams, similar to the C condition in Experiments 1 and 2); C-AC (concrete cover stories plus side-by-side concrete and abstract diagrams, similar to the AC condition in Experiments 1 and 2); and C-A (concrete cover stories plus abstract diagrams), the new condition.

From an experimental design point of view, it seems that our research should have used a 2×3 factorial design, with the first factor being the concreteness of the cover story (A or C) and the second factor being the concreteness of the diagram (A, C, or AC). However, practical reasons guided the omission of two possible combinations of factors, specifically, A-C and A-AC. We did not consider the condition A-C (abstract cover story and concrete diagram) or condition A-CA (abstract cover story and concrete combined with abstract diagram) because there is no clear one-to-one mapping from the abstract cover story elements to the real-life representations drawn in these two conditions. That is, unlike the other four conditions, a wide variety of instances of concrete diagrams may map to one abstract cover story, using the abstract engineering terminology for circuit components. Therefore, we decided against examining instructional conditions in which arbitrary concrete diagrams were provided for abstract cover story problems. Excluding the A-C and A-AC conditions permits a more rigorous comparison among the A-A, C-C, C-A, or C-AC conditions, which are also more pedagogically sensible because they provide students with a one-to-one rather than a one-to-many mapping of cover story and diagram representations.

Method

Participants and design. The participants were 96 college students (77 female and 19 male) enrolled in an educational psychology introductory course. Participants were given credit toward their final grade for their participation in the research. The mean age of the participants was 25.70 years ($SD = 9.38$ years). Of the students, 54 reported that they were Caucasian, 34 reported that they were Hispanic American, 2 reported that they were Native American, 1 reported that he or she was Asian American, and 5 reported that they had multiple or other ethnicities. There were 24 participants each in Groups A-A and C-AC, 23 participants in Group C-A, and 25 participants in Group C-C.

Materials and apparatus. The materials and apparatus were the same as those described in Experiment 1, with the following exception: We developed a fourth version of the instructional program to be used by the C-A group. This version was a combination of the A-A and C-C conditions. Specifically, Group C-A learned with the same contextualized cover stories presented to Group C-C and the same abstract representations of the electrical circuit elements presented to Group A-A. To promote the drawing of diagrams during problem solving, Experiment 3 explicitly prompted students to draw a diagram during the transfer test, as was done in Experiment 1.

Procedure. The procedure was identical to the one used in Experiment 1.

Results

The statistical analyses and tests were the same as those used in Experiment 1. Table 4 shows the mean scores and standard deviations for the four groups on all outcome measures. Similar to the first two experiments, no significant differences among the treatment groups were found on the pretest or on the time spent on the self-paced sections of the instructional program, $F(3, 92) = 1.56$, and $F(3, 92) = 0.46$, respectively.

Research Question 1: Does the type of diagram used during instruction affect problem-solving transfer? To answer this question, we first conducted an ANOVA with treatment condition as between-subjects variable and the practice session score as a dependent variable. The analysis did not reveal significant differences among the treatments, $F(3, 92) = 0.35$.

Second, we conducted an ANOVA with treatment condition as a between-subjects variable and the transfer test score as a dependent variable. The analyses showed a significant, medium-size effect, $F(3, 92) = 4.24$, $MSE = 44.33$, $p = .007$, $\eta^2 = .12$. Post hoc tests revealed that Group C-AC outperformed Groups C-C and C-A (both $ps < .05$) and that Group A-A outperformed Groups C-C and C-A ($p = .016$ and $p = .004$, respectively). No other significant differences were noted.

Research Question 2: Does the type of diagram used during instruction affect the quality and type of diagrams produced during problem solving? Thirty-four students (35.4%) attempted to draw at least one diagram during the posttest. A one-sample chi-square test showed that the percentage of participants that drew diagrams did not differ by condition, $\chi^2(3, N =$

Table 4
Means and Standard Deviations of Outcome Measures for Four Groups: Experiment 3

	Group							
	A-A		C-A		C-AC		C-C	
Measure	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
Practice time (min)	18.47_a	1.01	18.97_a	1.77	18.73_a	1.62	18.87_a	1.66
Pretest	1.29_a	1.80	2.04_a	2.22	2.58_a	2.64	2.52_a	2.61
Practice	4.45_a	1.25	4.17_a	1.61	4.54_a	1.47	4.24_a	1.45
Transfer	7.69_a	2.65	4.91_b	4.04	7.28_a	2.97	5.42_b	3.14
Diagram quality	24.10_a	10.46	24.50_a	8.35	22.90_a	8.67	16.60_a	8.59
A representations	5.60_a	4.88	6.25_a	3.77	6.40_a	3.17	1.20_b	1.99
C representations	0.00_a	0.00	0.00_a	0.00	0.60_a	1.26	3.20_b	3.36
Diagram perceptions	0.33_{ab}	0.48	0.13_a	0.34	0.29_{ab}	0.46	0.52_b	0.51

Note. A-A = abstract cover stories and diagrams; C-C = concrete cover stories and diagrams; C-AC concrete cover stories and abstract and concrete diagrams; C-A = concrete cover stories and abstract diagrams. Means in the same row that do not share subscripts are significantly different.

96) = 4.32, $p > .05$. To examine whether the type of diagram used during instruction affected the quality of diagrams that students produced during problem solving, we conducted an ANOVA with treatment condition as a between-subjects variable and students' graphic scores as a dependent measure. Similar to the prior two studies, there were no significant differences on this measure among treatments, $F(3, 92) = 1.25$.

To examine whether the type of diagram used during instruction affected the type of diagrams that students produced during problem solving, we first conducted an ANOVA with treatment condition as a between-subjects variable and the number of abstract representations produced in the transfer test as a dependent measure. The results revealed a large and significant treatment effect, $F(3, 30) = 4.41$, $MSE = 56.29$, $p = .01$, $\eta^2 = .31$. All groups that were presented with abstract diagrams during instruction (A-A, C-A, C-AC) produced significantly more abstract representations during the transfer test than Group C-C did ($ps = .01$, .02, and .003, respectively). In addition, an ANOVA with treatment condition as a between-subjects variable and the number of concrete representations produced in the transfer test as a dependent measure showed a large, significant treatment effect, $F(3, 30) = 5.48$, $MSE = 21.18$, $p = .004$, $\eta^2 = .35$. Group C-C produced significantly more concrete representations than Groups A-A, C-A, and C-AC ($ps = .001$, .01, and .06, respectively).

Research Question 3: Is there a correlation between the quality and type of diagrams produced and students' problem-solving transfer scores? As found in the prior two studies, a positive and significant correlation between students' diagram scores and their transfer scores was found (Pearson $r = .31$, $p = .002$). In addition, we found a nearly significant, positive correlation between the mean number of abstract representations drawn by the participants and their transfer test scores (Pearson $r = .31$, $p = .08$) but found no significant correlation between the number of concrete representations drawn by the participants and their transfer test scores (Pearson $r = -.05$, $p = ns$).

Research Question 4: Does the type of diagram used during instruction affect students' perceptions about the diagrams? To answer this question, we conducted an ANOVA with treatment condition as a between-subjects variable and the number of positive diagram comments as a dependent measure. A significant treatment effect was found, $F(3, 92) = 2.96$, $MSE = 0.62$, $p < .05$, $\eta^2 = .09$. Post hoc tests revealed that Group C-C produced significantly more positive comments about the program's diagrams than Group C-A ($p = .004$). No other significant differences were noted.

Discussion

The findings from Experiment 3 replicate the pattern of results found in Experiments 1 and 2 in the following ways. With respect to the transfer measure, the group that learned with simultaneous A and C diagrams outperformed the group that learned with C diagrams alone. Similar to Experiment 2, Group A-A (Group A in the prior experiments) outperformed Group C-C (Group C in the prior experiments). A new finding for this experiment was that both Groups C-AC and A-A (Groups AC and A in the prior experiments) outperformed the group that learned in the new condition (Group C-A). Therefore, the encoding-specificity hypothesis that practice with concrete problem stories during learning (encoding) should facilitate problem solving involving concrete problem stories (retrieval) as compared with practice with abstract problem stories was not supported.

In line with the prior findings, the groups that learned with abstract diagrams (A-A, C-A, C-AC) produced significantly more abstract diagrams during problem solving than the group that learned with concrete diagrams alone, and Group C-C drew significantly more concrete diagrams than the rest of the groups. Furthermore, students' diagram scores were positively correlated with their transfer scores across all studies, and the number of abstract diagram representations produced during problem solving was found to have a positive (albeit weak) association with students' transfer scores, whereas the number of concrete diagram representations was unrelated to students' transfer scores.

Unlike the prior two studies, however, Experiment 3 did not show a practice advantage for the group that learned with combined concrete and abstract diagrams over the groups that learned solely with concrete or abstract diagrams. This finding and the fact that the college students who participated in Experiment 3 showed descriptively higher practice scores than those of the high-school students who participated in Experiments 1 and 2 suggest that the benefits of combined concrete and abstract representations are especially important in helping precollege students connect their prior knowledge to the principles to be learned during scaffolded practice.

The transfer benefit of Groups C-AC and A-A over Group C-C found in Experiment 3 suggests that providing abstract diagrams during instruction can promote students' performance when asked to independently solve problems. However, this conclusion needs to be taken with caution because the new C-A treatment introduced in this study included abstract diagram representations, yet showed lower transfer performance than both the C-AC and A-A groups. How can we explain this discrepancy?

One interpretation of the low performance of Group C-A is the result of the mismatch between the concrete description of the cover stories used during instruction and the abstract nature of the diagrams used to represent such stories. The finding that Group C-A produced significantly fewer positive comments about the program diagrams than Group C-C seems to support this conclusion. However, this interpretation could also be used to argue that Group C-AC may also have experienced a mismatch, unless students actually used the concrete diagram before attempting to interpret and coordinate the abstract diagram. The finding that Group C-C performed poorly across all experiments suggests that it is the subsequent coordination of the two diagrams that has driven the observed benefits of Group C-AC. Yet, this thesis needs to be investigated in future research using appropriate methods, such as eye-tracking and think-aloud procedures.

It is important to note that the instructional program used in the present study did not explicitly teach students how to interpret the abstract diagrams included in the program. Students in Group C-A may have needed more guidance on mapping the abstract representations to the word problems. As noted in previous research, novices are unable to effectively use diagrams that make use of abstract conventions without appropriate experience (Petre & Green, 1993). It is possible that students in Group C-A would have produced higher transfer scores had they received appropriate support in mapping the concrete electrical circuit elements described by the cover stories and the corresponding abstract diagrams. A test of this interpretation is one of the directions of our future research.

General Discussion

Contemporary models of problem solving identify problem representation as one of the essential steps in the problem-solving process (Bruning, Schraw, Norby, & Ronning, 2004). Studies of both experts and novices show that the use of visual problem representations facilitates thinking and problem solving performance (Brenner et al., 1997; Chi et al., 1981; Collins & Ferguson, 1993; Mayer & Hegarty, 1996; Rittle-Johnson et al., 2001; Zhang, 1997). The effectiveness of visual representations, however, is highly dependent on the extent to which the different representations provide appropriate cognitive support (Butcher, 2006). The purpose of this research was to investigate the merits of using abstract and/or concrete diagrams during instruction to support the development of students' near-transfer problem solving.

Theoretical Implications

The theoretical contribution of the present study is to have tested the predictions that arise from the following three hypotheses: (a) concrete visual representations promote better problem solving by depicting a close correspondence between the representations and the concrete objects that they intend to represent; therefore, concrete visual representations rely less on knowledge conventions for their interpretations and help build connections between students' prior knowledge and the information to be learned (Bransford et al., 2000; Collins et al., 1989; Donovan & Bransford, 2005); (b) abstract visual representations promote better problem solving by focusing novice students' attention on structural rather than superficial problem information and by providing a tool to generalize the common underlying structure of problems that are superficially dissimilar (Bassok & Holyoak, 1989; Butcher, 2006; Goldstone & Son, 2005; Joseph & Dwyer, 1984; Sloutsky, Kaminski, & Heckler, 2005); and (c) simultaneous abstract and concrete visual representations promote better problem solving by combining the cognitive advantages of both representation types (Scheiter et al., 2009).

Across the three studies, we found several key regularities in support of the predictions derived from the third hypothesis. Experiments 1 and 2 showed that the transfer of the principles learned during instruction to practice was greater when the problems were presented with both abstract and concrete representations of the cover stories. When asked to independently solve a set of problems with no supporting diagrams, Group AC outperformed the participants who learned with concrete diagrams alone across all experiments and outperformed the group in which students learned with identical cover stories but only abstract diagrams in Experiment 3. Although we expected to find a parallel treatment effect on students' transfer and diagram quality, this was not the case. A possible interpretation for this discrepancy is that the treatments were not specifically designed to promote better diagram representations but rather to use the provided diagrams to solve problems.

Similar to other studies, we found positive, moderate correlations between students' problem representations and their problem-solving scores across the three studies (Brenner et al., 1997; Collins & Ferguson, 1993; Mayer & Hegarty, 1996; Rittle-Johnson et al., 2001; Zhang, 1997). A hypothesis to be tested in the future is that the correlation between students' visual representations and problem-solving scores is higher when students are provided with explicit, problem representation instruction. In addition, an interesting finding that needs further research is the positive correlation between students' use of abstract representations and transfer across the three studies. This finding is in line with past research showing that the use of schematic (abstract) representations is associated with mathematical problem-solving success, whereas the use of pictorial (concrete) representations is negatively correlated with problem-solving success (Edens & Potter, 2007, 2008; Hegarty & Kozhevnikov, 1999; van Garderen & Montague, 2003).

A finding that is out of the scope of our research questions yet interesting is that the percentage of high-school students who drew diagrams during the problem-solving transfer test dropped from 84.51% (Experiment 1) to 14.84% (Experiment 2) when students were not explicitly instructed to draw diagrams and that only 35.42% of the college students who participated in Experiment 3 attempted to draw problem representations during problem solving despite being explicitly prompted to do so. We believe that these results are consistent with past research showing that novices, unlike experts, underuse visual representations during problem solving (Kozma, 2003). Nevertheless, the low response on the drawing task embedded in the problem-solving test is an interesting finding that deserves future investigation. For instance, it is not clear whether this low response was the result of students' lack of compliance, their belief that the task was not useful to achieve the goal of solving the problem, their lack of drawing self-efficacy, or the fact that our treatments were not specifically designed to train students in diagram drawing. Because the treatments used in the present research did not focus on promoting the drawing of visual representations, a potential area for future research is to investigate methods that can help novice students become better producers and consumers of problem representations.

Taken together, the findings of this research suggest that the design of problem representations should be guided by the learning objectives of the lesson. When problem representations do not communicate relevant aspects of the information to be learned, these representations can hinder rather than foster learning. Evidence in support of this conclusion comes from the underperformance of the group that learned with concrete diagrams alone. This *irrelevant concreteness* has been argued to prevent deeper conceptual processing and the extraction of underlying relational commonalities among superficially different problems (Sloutsky et al., 2005). For instance, the different concrete instantiations of resistive elements in a circuit (e.g., lightbulbs or motors) and the concrete shapes of the real-life depictions of the battery or the lightbulbs and motors are irrelevant to the underlying electrical function of the battery as a voltage source and the lightbulbs and motors as resistors. Yet, the assumption that the use of concrete representations can, in and of itself, promote learning has been highly advocated in education (Goldston & Son, 2005). One of the contributions of this research is to have empirically tested this strong assumption with a set of carefully designed studies. Specifically, the findings suggest that the intuitively appealing case for using perceptually rich, concrete problem representations should be taken with caution. Because perceptually rich representations have limited referential flexibility, they should be presented with corresponding abstract representations to promote problem-solving transfer (DeLoache, 2000; Goldstone & Sakamoto, 2003). A difficulty found when teaching with concrete representations is that novices typically do not find a connection between the representation and the desired principles to be transferred (Clements & McMillen, 1996; Uttal, Liu, & DeLoache, 1999). Our results suggest that problem solving is best supported when learners are eventually able to produce and use abstract visual representations to support the problem-solving process but that these representations should be scaffolded by concrete visual representations that connect their prior knowledge with the to-be-learned information. However, additional research is necessary to test this hypothesis because neither students' interpretations of the diagrams nor their connections to prior knowledge were tested directly in our studies.

Practical Implications

In recent years, the pedagogical use of tutoring systems, such as the computer-based program used in the present study, has grown dramatically (Koedinger & Aleven, 2007; Ritter, Anderson, Koedinger, & Corbett, 2007). Yet, there is still a deep need for more systematic research that explores the potential educational impact of the many characteristics of these tools. Our research is an attempt to address one component of this pursuit. The reported experiments revealed that a combination of both concrete and abstract problem representations during instruction best supported students' problem-solving transfer.

In addition, this research suggests practical implications for engineering education. A review of introductory electrical engineering textbooks shows that those designed for precollege students (such as the participants in Experiments 1 and 2) typically present concrete, real-world problems and representations (Orsak et al., 2004). In contrast, college-level textbooks mostly rely on abstract problem representations (Alexander & Sadiku, 2004; Irwin & Nelms, 2005). To our knowledge, the research base for the assumptions underlying these different instructional designs is lacking. Although the findings of the present study are preliminary and need to be replicated with different students and engineering topics, they suggest that precollege engineering instruction may benefit from combined concrete and abstract problem representations. According to cognitive developmental theories, this practical implication is appropriate because precollege students are able to engage in abstract thinking (Meece, 2002; Miller, 2002).

Limitations

A theoretical limitation of the present study is that we did not investigate the effects of learning with abstract cover stories and concrete diagrams (A-C) or with abstract cover stories and a combination of abstract and concrete diagrams (A-AC). Although these two instructional conditions are not found in current textbooks and present students with the issue of having to map arbitrary concrete diagram elements to make sense of the abstract cover story description, having included them in our design would have strengthened our conclusions. For instance, the results of Experiment 3 do not address whether it is the format of the cover story or the match between the visual representation and cover story formats that produced the learning benefits of Group A-A over Group C-A. An answer to this issue would require testing the A-C condition. Further, having examined the A-AC condition would have helped test the hypothesis that a combination of abstract and concrete visual representations is always more supportive of students' problem solving, regardless of the concreteness of the cover story.

The present research is also limited because the experimental conditions in which the participants learned cannot be generalized to learning from other more authentic situations in which students spend several days learning with materials that are embedded in their curriculum (Bransford & Schwartz, 1999). An additional limitation stems from having set as our main learning measure a paper-and-pencil near-transfer task. Because one of the advantages of realistic representations is their high resemblance with the real-world objects that they depict, future studies should examine the role of concrete representations when the learning objective requires far transfer (i.e., the solution of problems with a different underlying structure than the training problems or problem solving with realistic objects in natural settings; Bétrancourt & Tversky, 2000).

Moreover, our research provides evidence that simultaneously using concrete and abstract problem representations during instruction promotes problem-solving success. What our research has not addressed is whether the simultaneous presentation of abstract and concrete representations is better than a sequential presentation of the representations. For example, a cognitive tutor study has shown that promoting inductive reasoning from prior perceptual experience before asking students to produce an algebraic expression can promote algebra learning (Koedinger & Anderson, 1998) and Nathan (1998; Nathan, Kintsch, & Young, 1992) found that novice word-problem solvers' overreliance on concrete situation models can be avoided by initially introducing students to the concrete situation and subsequently encouraging them to reason about the situations. Therefore, novice students may benefit from initial concrete instruction, which can later be faded out as students are given guidance about how to represent the elements included in any cover story in an abstract fashion (Goldstone & Son, 2005; Scheiter et al., 2009). Future research should examine this promising instructional method.

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Received December 20, 2009
Revision received October 4, 2010
Accepted October 27, 2010 ■